

> restart;

> read `/M/m/findrel_v2.7`:

> f := u(n+2) = (n+2)*((9*n+15)*u(n) - (6*n+28)*u(n+1))/(n+7)/(3*n+2);

$$f := u(n+2) = \frac{(n+2) ((9n+15)u(n) - (6n+28)u(n+1))}{(n+7)(3n+2)}$$

> findrel(f, u(n), {[0,0,0,1], u(0) = 0, u(1) = 1});

$$u(n) = 3(-1)^n \left(-\frac{(n^2+3n+2)A001006(n)}{(n+4)(n+5)} - \frac{(n^2+3n-2)A001006(n+1)}{(n+4)(n+5)} \right), 1 \leq n$$

> Our_sequence_without_hg := rhs(%[1])/(-1)^n;

$$\text{Our_sequence_without_hg} := -\frac{3(n^2+3n+2)A001006(n)}{(n+4)(n+5)} - \frac{3(n^2+3n-2)A001006(n+1)}{(n+4)(n+5)}$$

> for i to 2 do 'A001006'(n+1+i) = collect(A001006(n+1+i), A001006, factor) od;

$$A001006(n+2) = \frac{3(n+1)A001006(n)}{n+4} + \frac{(2n+5)A001006(n+1)}{n+4}$$

$$A001006(n+3) = \frac{3(2n+7)(n+1)A001006(n)}{(n+5)(n+4)} + \frac{(7n^2+42n+59)A001006(n+1)}{(n+5)(n+4)}$$

> result := collect(x*A001006(n+3) + y*A001006(n+1) + z*A001006(n), A001006, factor);

$$\text{result} := \frac{(6xn^2+27xn+21x+zn^2+9zn+20z)A001006(n)}{(n+5)(n+4)} + \frac{(7xn^2+42xn+59x+yn^2+9yn+20y)A001006(n+1)}{(n+5)(n+4)}$$

> solve({numer(coeff(result, A001006(n))) = -3*(n+2)*(n+1), numer(coeff(result, A001006(n+1))) = -3*(n^2+3*n-2)}, {x,y,z});

$$\left\{ z = -\frac{3(2xn^2+9xn+7x+2+n^2+3n)}{n^2+9n+20}, y = -\frac{7xn^2+42xn+59x-6+3n^2+9n}{n^2+9n+20}, x = x \right\}$$

> assign(%);

> x := a*n+b;

$$x := an + b$$

> rem(-3*(7*x*n^2+42*x*n+59*x-6+3*n^2+9*n), (9*n+n^2+20), n);

rem(-3*(2*x*n^2+9*x*n+7*x+2+n^2+3*n), (9*n+n^2+20), n);

$$(-324a + 54 + 63b)n + 198 - 1260a + 243b$$

$$(-144a + 18 + 27b)n + 54 - 540a + 99b$$

> solve(%, %), {a,b});

$$\{a = -1, b = -6\}$$

> assign(%);

> assign(('x','y','z') = (op(simplify([x,y,z]))));

> collect(x*A001006(n+3) + y*A001006(n+1) + z*A001006(n),

```
A001006, factor):
```

```
> is(% = Our_sequence_without_hg); # recall that we only stripped  
off (-1)^n
```

true

And so, since A001006 is always integer, $u(n)$ is the sum of products of integers:

```
> u(n) = (-1)^n*(x*'A001006(n+3)' + y*A001006(n+1) + z*A001006(n)  
), n>=1;
```

$$u(n) = (-1)^n ((-n-6) A001006(n+3) + (7n+18) A001006(n+1) + (6n+6) A001006(n)), 1 \leq n$$

```
>
```