1. Let A be an n by n matrix and let v be some vector. How to check if v is an eigenvector?
   Answer: If v is zero then v is *not* an eigenvector. If v is not zero, compute Av and check if Av is a scalar multiple of v. If so (if $Av = \lambda v$ for some scalar λ) then v is an eigenvector, with eigenvalue λ.

2. Let λ be some scalar. What does it mean when we say that λ is an eigenvalue of matrix A?
   Answer: This means that there exists a *non-zero* vector v for which Av equals λv. So that vector v is very special because multiplying it by matrix A has the same effect as multiplying it by this scalar λ.

3. If I know an eigenvalue λ (so λ here is some known number) then how do I compute the corresponding eigenvectors?
   Answer: We have to solve $Av = \lambda v$ and this is the same thing as solving $Mv = 0$ where matrix M equals $A - \lambda I$. Thus, compute $A - \lambda I$, and compute a basis of the Nullspace of that. Your basis of $\mathrm{Nul}(A - \lambda I)$ is what you would write down as your answer.
   Note: every linear combination (except 0) of these basis vectors would be an eigenvector. But to describe all these eigenvectors, it is sufficient to write down just a basis of $\mathrm{Nul}(A - \lambda I)$, because from that basis it is easy to get all other eigenvectors with eigenvalue λ.

4. What if I row-reduce $A - \lambda I$ and I don't get any free variable(s)?
   Answer: There are two possibilities. Either λ is not an eigenvalue (which means that when you computed the eigenvalues (see the item below) you made a computation error) or you made an error when you computed $A - \lambda I$ or when you row-reduced it.

5. How do we find the eigenvalues?
   Answer: In all of the above items, the number λ was assumed to be a *known* number. But, if we do not yet know the eigenvalues, then we must treat λ as an *unknown* number. What you do is this:
   1) Compute matrix $A - \lambda I$ where λ is an unknown.
   2) Compute the determinant of that matrix $A - \lambda I$. The result will be a polynomial in your unknown λ of degree n when A is n by n. Equate that polynomial $\det(A - \lambda I)$ to zero. That gives you an equation for your unknown λ. This equation is called the *characteristic equation.*
   3) Solve the characteristic equation. This way you find all numbers λ for which $A - \lambda I$ has determinant 0. Hence, by solving the character-

istic equation you find all numbers λ for which row-reducing $A - \lambda I$ leads to free variable(s) (see item 4) and $\mathrm{Nul}(A - \lambda I)$ contains non-zero vectors (see item 3).

6. Summary. How do I compute the eigenvectors?
   Answer: If you already know the eigenvalues, then do item 3 for each eigenvalue.
   If you don't already know the eigenvalues, then let λ be an unknown. Compute $A - \lambda I$ so this is a matrix and the unknown λ appears on the diagonal (with minus signs). Then compute the determinant. Equate that to zero and you have the characteristic equation. Solve, and you have the eigenvalues. Then do item 3 for each eigenvalue (also remember item 4).

7. Checks. Once you found the characteristic polynomial and the eigenvalues: *check your computation!!!*. If you made an error here, you can end up with very complicated (but wrong) eigenvalues and you may end up with a very long computation that fails at the end (see item 4).

   Once you found the eigenvectors: Computing eigenvectors is a lot of work. Checking eigenvectors is little work, so check each eigenvector (see item 1 on how to do that).
   What if you discover an error this way but you don't have enough time left to recompute everything (characteristic equation, eigenvalues, eigenvectors)? Well, you get more partial credit for "wrong eigenvector + check" than for "wrong eigenvector + no check".