Linear algebra, test 3.

March 17, 2000

Write down your name and SSN.

1. Let

$$u = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 2 \end{pmatrix} \quad \text{and} \quad v = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 2 \end{pmatrix}$$

   (a) (10 points). What is $\cos(\theta)$ where θ is the angle between u and v? What is the length of u and v?

   (b) (10 points). What is the vector projection of u onto v, and what is the length of this projection?

   (c) (15 points). Let $A = (u \ v)$. Compute a matrix B such that the null space of B is the column space of A

$$\mathcal{CS}(A) = \mathcal{NS}(B).$$

   (d) (10 points). For which values of x and y is the vector

$$w = \begin{pmatrix} x+1 \\ x \\ y \\ -y \end{pmatrix}$$

   an element of SPAN($\{u, v\}$)?

2. (25 points) Consider the following vectors

$$u = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

   Is u as a linear combination of v_1, v_2, v_3? If so, give that linear combination and verify that your answer is correct.

3. Let

$$A = \begin{pmatrix} 2 & 2 & -2 \\ -1 & 1 & 1 \\ 0 & 2 & 0 \end{pmatrix}.$$

    (a) (10 points) Compute the characteristic polynomial of A.

    (b) (5 points) Compute the eigenvalues of A. Use the trace and the determinant of A as a way to check the correctness your answer.

    (c) (15 points) For each eigenvalue, compute one corresponding non-zero eigenvector.

    (d) (10 bonus points). Write the vector

$$w = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

as a linear combination of these three eigenvectors. Use that linear combination to calculate $B \cdot w$ where $B = A^{10}$ without doing a lot of matrix multiplications.

Good luck!