Linear algebra, test 2.

February 22, 2001

Do not forget to write down your name!

1. Let

$$A = \begin{pmatrix} 0 & 1 & a \\ 1 & 0 & 1 \\ 1 & 2 & 3 \end{pmatrix}.$$

   (a) (10 points). Calculate the determinant of A.

   (b) (10 points). Let

$$K = \begin{pmatrix} a \\ 1 \\ 1 \end{pmatrix}, \quad \text{and} \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

   If $AX = K$ then compute the value of x_1 using Cramer's rule. You do not need to compute the values of x_2 and x_3.

   (c) (2 points). Is there any value for a such that the equation $AX = K$ from the previous question does not have a solution? If so, what is that value for a?

2.

$$A = \begin{pmatrix} 1 & -3 \\ 6 & -8 \end{pmatrix}$$

   (a) (15 points). Write A as a product of elementary matrices.

   (b) (2 points). Compute the determinant of A.

   (c) (8 points). Compute the characteristic polynomial of A.

   (d) (10 points). Compute the eigenvalues of A and for each eigenvalue compute a corresponding eigenvector.

   (e) (3 points). Can you predict what the eigenvalues of matrix $B = A^2 + 7A$ are *without calculating matrix* B?

3.

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 2 & -2 \\ -1 & 4 & 1 \end{pmatrix}$$

(a) (10 points). Compute an LU factorization of A.

(b) (2 points). Compute the determinant of A.

4. Let

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 1 \end{pmatrix}.$$

(a) (10 points) Compute $\mathrm{Adj}(A)$, the adjoint matrix of A.

(b) (5 points) Check your answer by computing $A \cdot \mathrm{Adj}(A)$ and the determinant of A.

5. (a) (5 points). What is the area of the triangle that has the following three points as vertices: $(0,0)$, $(2,5)$, $(4,3)$.

   (b) (2 points). True or false: $\det(AB) = \det(A)\det(B)$ for all n by n matrices A and B.

   (c) (2 points). True or false: $\det(A+B) = \det(A) + \det(B)$ for all n by n matrices A and B.

   (d) (2 points). True or false: $\det(A^T) = \det(A)$ for all n by n matrices A (recall that the A^T is the transpose of A).

   (e) (2 points). True or false: $\det(ABC) = \det(CBA)$ for all n by n matrices A, B and C.

Good luck!