Linear algebra, test 3

March 22, 2001

Write down your name and SSN.

1. (35 points) Let

$$A = \begin{pmatrix} 1 & 1 & -1 & -1 \\ 5 & 6 & -1 & 0 \\ 0 & 1 & 4 & 5 \end{pmatrix}.$$

(a) Compute the reduced row echelon form of A and the rank of A.

(b) Compute a basis B for the column space $CS(A)$.

(c) Compute the coordinate vector (with respect to B) of each column of A.

(d) Compute a basis for the null space $\mathcal{NS}(A)$.

(e) Give a linear relation between the columns of A.

(f) Find a matrix C such that $NS(C) = CS(A)$.

(g) Find (if it exists) a non-zero vector $v \in \mathbb{R}^3$ that is orthogonal (dotproduct 0) to every column of A.

2. Let A be the matrix $A = (1\ 1\ 1\ 1)$. Let $V = NS(A)$. Let

$$u_1 = \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}, u_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}, u_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}, u_4 = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}, u_5 = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

$$u_6 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}, u_7 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

(a) (5 points) Which (if any) of the vectors $u_1, \ldots, u_7$ is NOT in V?

(b) (7 points) If M is a matrix with m rows, n columns, and rank r, then give a formula for the dimension of the null space of M.

(c) (8 points) Compute the rank of A, and compute the dimension of V with this formula.

(d) (15 points) Find out which of the following sets are a basis for V:
$\{u_1\}$
$\{u_1, u_2\}$
$\{u_1, u_2, u_3\}$
$\{u_1, u_2, u_3, u_4\}$
$\{u_1, u_2, u_4\}$
$\{u_1, u_5\}$
$\{u_1, u_2, u_5\}$
$\{u_1, u_2, u_4, u_5\}$
$\{u_1, u_6\}$
$\{u_1, u_2, u_6\}$
$\{u_1, u_2, u_4, u_5, u_6\}$.
$\{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$.

(e) (5 points) Which of these sets are not a basis of V but just a spanning set for V?

(f) (5 points) Which of these sets are not a basis but are still linearly independent?

(g) (5 points) Let w be the vector

$$w = \begin{pmatrix} x \\ x^2 \\ x \\ x+2 \end{pmatrix}$$

where x is some unknown number. For which values of x would w be an element of V?

3. Let V be a subspace of $\mathbb{R}^5$ with as basis $B = \{u_1, u_2\}$ where

$$u_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, u_2 = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}.$$

Let

$$v_1 = \begin{pmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 3 \\ 5 \\ 7 \\ 9 \\ 11 \end{pmatrix}, v_3 = \begin{pmatrix} 2 \\ 3 \\ 5 \\ 7 \\ 11 \end{pmatrix}$$

(a) (10 points) Is $v_1 \in V$? If so, then calculate $[v_1]_B$.
Same question for v_2 and v_3.

(b) (5 points) Let $W = \text{SPAN}(v_1, v_2, v_3)$.
Is $V \subseteq W$?
Is $W \subseteq V$?
Is $V = W$?

Good luck!