Linear algebra, test 2.

1.

$$A = \begin{pmatrix} 2 & 2 \\ 2 & 1 \end{pmatrix}$$

(a) Write A as a product of elementary matrices.

(b) Compute A^{-1}

(c) Suppose that $AX = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. What is X?

(d) Compute an LU factorization of A.

2. Let

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 2 & 3 \end{pmatrix}$$

(a) Compute the inverse of A.

(b) Write down the transpose of A.

3. Let

$$u = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad v = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \quad \text{and} \quad w = \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}.$$

Let $A = (u \;\; v \;\; w)$. Compute the following (hint: you can answer all of these by rowreducing just one matrix (AX) where X is a column vector with x_1, x_2, x_3, x_4 as entries.

(a) A basis of the column space of A.

(b) A basis of the null space of A.

(c) Are the columns of A linearly dependent? If so, then write down a linear relation.

(d) A matrix B such that the null space of B is the column space of A.

(e) A basis of the null space of B.

4. Consider the linear map $T : \mathbb{R}^3 \mapsto \mathbb{R}^2$ given by

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_3 \\ x_2 - 2x_3 \end{pmatrix}.$$

(a) Write down the matrix of this linear map.

(b) How can we see from this matrix if T is 1-1 or not, and how can we see from this matrix if T is onto or not?

(c) Is T 1-1?

(d) Is T onto?