Linear algebra, sample questions, turn in Monday for extra credit on test 3

March 18, 2004

1. Let

$$u_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad u_2 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \quad v_1 = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix}, \quad v_3 = \begin{pmatrix} -1 \\ 0 \\ 5 \end{pmatrix}.$$

(a) Let $V = \text{SPAN}(\{u_1, u_2\})$. Show that $B := \{u_1, u_2\}$ is a basis of V.
A hint that won't be on the test: In general, you'd have to check two things:

(a) Is $\{u_1, u_2\}$ a spanning set of V? (answer is clearly yes because V is the SPAN of u_1, u_2).
(b) Is $\{u_1, u_2\}$ linearly independent?

Both would have to be "yes" in order for the final answer (basis yes or no) to be yes.

(b) Can you find some matrix such that V is the column space of that matrix?

(c) Can you find some matrix such that V is the Null space of that matrix?

(d) Are v_1, v_2, v_3 in V? If so, express v_1, v_2 and v_3 as linear combinations in u_1 and u_2.

(e) Give $[v_1]_B$, $[v_2]_B$, and $[v_3]_B$.
(these are the coordinate vectors of v_1, v_2, v_3 with respect to B, see section 4.4 in the book).

(f) Is $\{v_1, v_2, v_3\}$ a spanning set of V?
A hint that will not be given on the actual test: In general, to check if $\{v_1, v_2, v_3\}$ is a spanning set of V you'd have to check the following two things:

(a) Are v_1, v_2, v_3 actually in V? (was a previous question)
(b) Is every element of V a linear combination of v_1, v_2, v_3?

Both would have to be "yes" in order for the answer to the question to be "yes". Now as to checking (b), since V is the SPAN of u_1, u_2, all you'll need to check is that each of u_1, u_2 is in the SPAN of $\{v_1, v_2, v_3\}$. If they both are, then the answer to (b) is "yes", otherwise it is "no".

(g) Is $\{v_1, v_2, v_3\}$ a basis of V?
Again the hint: In general, you'd have to check two things:

(a) Is $\{v_1, v_2, v_3\}$ a spanning set of V? (see previous question).
(b) Is $\{v_1, v_2, v_3\}$ linearly independent?

Both would have to be "yes" in order for the final answer (basis yes or no) to be yes.

(h) Is some subset of $\{v_1, v_2, v_3\}$ a basis of V?
(in other words: can we throw away some element(s) so that the remaining elements form a basis of V)?

2. A hint that won't be on the test: A set $u_1, \ldots, u_p \in \mathbb{R}^n$ is a basis of $\mathbb{R}^n$ if and only if the matrix $(u_1 \cdots u_p)$ is an invertible n by n matrix. If $p \neq n$ then $u_1, \ldots, u_p$ can never be a basis of $\mathbb{R}^n$, but if $p = n$ then to check if $u_1, \ldots, u_p$ is a basis of $\mathbb{R}^n$ we would have to check if matrix $(u_1 \cdots u_p)$ is invertible. We can do check if it is invertbile either by computing the determinant (matrix invertible when determinant $\neq 0$) or by row-reducing (matrix invertible when it row-reduces to the identity matrix I. We don't have to row-reduce all the way to I though, we can stop earlier. The matrix is invertible when it is square and you don't get any zero-rows in the row-echelon form (it is not necessary to go all the way to *reduced* row-echelon form because if don't get zero-rows when you row-reduce a square matrix then the reduced row-echelon form can only be I)).

(a) Do the following three vectors form a basis of $\mathbb{R}^3$?

$$u_1 = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}, \quad u_2 = \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix}, \quad u_3 = \begin{pmatrix} -1 \\ 0 \\ 5 \end{pmatrix}.$$

(b) Do the following three vectors form a basis of $\mathbb{R}^3$?

$$u_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad u_2 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \quad u_3 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}.$$

(c) Do the following three vectors form a basis of $\mathbb{R}^3$?

$$u_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \quad u_2 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \quad u_3 = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}.$$