Linear algebra, test 3.

March 25, 2004

1. Let

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 5 & 7 \end{pmatrix}$$

(a) (5 points). Compute the reduced row echelon form of $A$.
(b) (8 points). Write down a basis $B$ of $\text{Col}(A)$.
(c) (12 points). Give a basis $C$ of $\text{Nul}(A)$.
(d) (10 points). Let $v_1, v_2, v_3, v_4$ be the columns of matrix $A$. What are: $[v_1]_B$, $[v_2]_B$, $[v_3]_B$ and $[v_4]_B$.
(e) (10 points). Let

$$w = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

What’s the easiest way to tell if $w$ is in $\text{Nul}(A)$ or not?

If $w$ is in $\text{Nul}(A)$, then compute its coordinate vector $[w]_C$ with respect to the basis $C$ you computed in question (c).
2. Let \( u = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \) and \( v = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \). Let \( V = \text{SPAN}\{u,v\} \) and \( B = \{u,v\} \) a basis of \( V \).

(a) (5 points). If \( w \) is some vector in \( V \) for which \( [w]_B = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \) then what is \( w \)?

(b) (15 points). Which of the following vectors are in \( V \)? For each that is in \( V \), give the coordinate vector with respect to \( B \).

\[
\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ 5 \\ 4 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}.
\]

Question 2 continues on the next page.
(c) (3 points). Give a matrix $A$ for which: $V = \text{Col}(A)$.
(d) (12 points). Give a matrix $N$ for which: $V = \text{Nul}(N)$.
(e) (2 points). True or false: The vectors in question (b) form a spanning set for $V$?
(f) (2 points). True or false: If we take those vectors in question (b) that were in $V$, then we get a spanning set for $V$?
(g) (2 points). True or false: If we take those vectors in question (b) that were in $V$, then we get a basis for $V$?
3. (10 points). For each of the following, mention if it is a vector space or not. 
If it is not a vector space, then say why it is not a vector space by writing one of the following three things: "does not contain the zero vector", or "is not closed under addition", or "is not closed under scalar multiplication". 
If it is a vector space, then no further explanation will be necessary. 

(a) \( V = \{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \} \).

(b) \( V = \{ \begin{pmatrix} a \\ b \end{pmatrix} \mid ab = 0 \} \).

(c) \( V = \{ \begin{pmatrix} a \\ b \end{pmatrix} \mid a + b + 3 = 0 \} \).

(d) \( V = \{ \begin{pmatrix} a \\ b \end{pmatrix} \mid 2a - 5b = 0 \} \).

(e) \( V = \{ \begin{pmatrix} a \\ b \end{pmatrix} \mid a \geq b \} \).

4. (5 points). If \( B = \{ t^2 + t, t + 1, 1 \} \) then what is \([t^3 - t + 1]_B\)?