MAS 3301 Modern Algebra Homework Set 2

1. Simplify each expression, rewriting in the form $a+bi$, where a and b are real numbers.

   (i) $\dfrac{1+2i}{3-4i}+\dfrac{2-i}{5i}$ *Ans.* $-\frac{2}{5}$

   (ii) $\dfrac{5}{(1-i)(2-i)(3-i)}$

   (iii) $(1-i)^4$

2. Verify that each of the two numbers $z=1\pm i$ satisfies the equation $z^2-2z+2=0$.

3. In each case, sketch in the complex plane the set of complex numbers determined by the given condition.

   (i) $|z-1+i|=1$; (ii) $|z+i|\le 3$; (iii) $\mathrm{Re}(\overline{z}-iz)=2$; (iv) $|2z-i|=4$.

4. By setting $z=a+bi$ and $w=c+di$, verify by calculation that

   (i) $\overline{z\pm w}=\overline{z}\pm\overline{w}$; (ii) $\overline{zw}=\overline{z}\,\overline{w}$; (iii) $\overline{\left(\dfrac{z}{w}\right)}=\dfrac{\overline{z}}{\overline{w}}$.

5. By writing the individual factors on the left of each equation in polar form, performing the needed operations, and finally changing back to rectangular coordinates, verify that

   (i) $i(1-\sqrt{3}i)(\sqrt{3}+i)=2(1+\sqrt{3}i)$; (ii) $5i/(2+i)=1+2i$;

   (iii) $(-1+i)^7=-8(1+i)$; (iv) $(1+\sqrt{3}i)^{-10}=2^{-11}(-1+\sqrt{3}i)$.

6. Use de Moivre's formula to derive the following trigonometric identities:

   (i) $\cos 3\theta=\cos^3\theta-3\cos\theta\sin^2\theta$; (ii) $\sin 3\theta=3\cos^2\theta\sin\theta-\sin^3\theta$.

7. If you have not done so already, work problem 6 from the first homework set.

8. Each of the following numbers is algebraic. For each, find a polynomial with integer coefficients that has that number as a root.

   (i) $\dfrac{\sqrt[3]{7}}{3}$; (ii) $\dfrac{\sqrt[3]{7}}{\sqrt{2}}$; (iii) $\sqrt[3]{7}+1$; (iv) $\sqrt{2}+\sqrt{3}$; (v) $\sqrt{2}+\sqrt[3]{3}$.