Test 1, Feb 13 2005, MAS3301

1. (10 points). Use the Euclidean algorithm to find two integers s, t for which $129s + 160t = 1$.

2. (10 points). Find the two complex solutions $a \pm bi$ of the equation $x^2 + x + 1 = 0$.

3. (10 points). Write $1 + i$ in polar coordinates: $1 + i = re^{i\alpha}$ where the real numbers r, α are $r = \dots$ and $\alpha = \dots$.

4. (10 points). Find a complex number written in polar coordinates $z = re^{i\alpha}$ for which $z^8 = -1$ (it is enough to give just one such z).

5. (10 points). Which field axiom(s) is/are not satisfied by the quaternions?

6. (10 points). Make a sketch of all z in the complex plane for which $\overline{z} = z - i$.

7. (10 points). Find a polynomial with integer coefficients that has $3 + 2\sqrt{2}$ as a root.

8. (10 points). The set of non-negative rational numbers $\{a \in \mathbf{Q} \mid a \geq 0\}$, is this a field? If not, which field axiom(s) does this set not satisfy?

9. (10 points). $e^{i\pi/4} = \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{2}i$

   Use this fact to compute the following: Let v be the vector $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Rotate this vector by an angle $\pi/4$ (counter clockwise). What is the result?

10. Let z be the quaternion $1 + i + j + k$.

    (a) (2 points). What is the absolute value of z?

    (b) (2 points). What is the conjugate of z?

    (c) (2 points). Compute z^{-1}, the multiplicative inverse of z.

    (d) (2 points). Compute the product zi.

    (e) (2 points). Compute ziz^{-1}.