Sample questions

1. Find (use the Euclidean algorithm) two integers s, t for which $145s + 121t = 1$.
2. Explain why there do not exist integers s, t for which $35s + 45t = 1$.
3. Find a complex number $z = a + bi$ whose square is $z^2 = 2i$ (note: there are two correct answers, it suffices to give just one of them).
4. Write i in polar coordinates: $i = re^{i\alpha}$ where $r = \ldots$ and $\alpha = \ldots$.
5. Find a complex number written in polar coordinates $z = re^{i\alpha}$ for which $z^4 = i$ (again, it suffices to give just one such z).
6. What is the real part of the complex number $e^{i\pi/5}$?
7. Which field axiom(s) is/are not satisfied by the dual numbers (the dual numbers are of the form $a + b\epsilon$ where a, b are real numbers, and ϵ satisfies the relation $\epsilon^2 = 0$).
8. In the dual numbers, find the multiplicative inverse of $2 + \epsilon$.
9. In the quaternions, find the multiplicative inverse of $1 + i + j$.
10. Make a sketch of all z in the complex plane for which $\mathrm{Re}(z) = \mathrm{Im}(z)$.
11. Make a sketch of all z in the complex plane for which $|z - 1| = 1$.
12. Let $\alpha = \sqrt[3]{2}$. Find a polynomial with integer coefficients that has $\alpha^2 - \alpha$ as a root.
13. The set of all even integers $E = \{\ldots, -4, -2, 0, 2, 4, \ldots\}$, which of the field axiom(s) does this set not satisfy?
14. $e^{i\pi/6} = \frac{1}{2}\sqrt{3} + \frac{1}{2}i$

    Use this fact to compute the following: Let v be the vector $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Rotate this vector by an angle $\pi/6$ (counter clockwise). What is the result?
15. If z is a non-zero complex number, and if $\overline{z}$ happens to be equal to z^{-1}, then the absolute value of z must be:
16. Compute the quaternion $(i + j - k) \cdot (1 - i - j)$.