Sample questions

1. Find (use the Euclidean algorithm) two integers $s, t$ for which $145s + 121t = 1$.

2. Explain why there do not exist integers $s, t$ for which $35s + 45t = 1$.

3. Find a complex number $z = a + bi$ whose square is $z^2 = 2i$ (note: there are two correct answers, it suffices to give just one of them).

4. Write $i$ in polar coordinates: $i = re^{i\alpha}$ where $r = \ldots$ and $\alpha = \ldots$.

5. Find a complex number written in polar coordinates $z = re^{i\alpha}$ for which $z^4 = i$ (again, it suffices to give just one such $z$).

6. What is the real part of the complex number $e^{i\pi/5}$?

7. Which field axiom(s) is/are not satisfied by the dual numbers (the dual numbers are of the form $a + b\epsilon$ where $a, b$ are real numbers, and $\epsilon$ satisfies the relation $\epsilon^2 = 0$).

8. In the dual numbers, find the multiplicative inverse of $2 + \epsilon$.

9. In the quaternions, find the multiplicative inverse of $1 + i + j$.

10. Make a sketch of all $z$ in the complex plane for which $\text{Re}(z) = \text{Im}(z)$.

11. Make a sketch of all $z$ in the complex plane for which $|z - 1| = 1$.

12. Let $\alpha = \sqrt{2}$. Find a polynomial with integer coefficients that has $\alpha^2 - \alpha$ as a root.

13. The set of all even integers $E = \{\ldots, -4, -2, 0, 2, 4, \ldots\}$, which of the field axiom(s) does this set not satisfy?

14. $e^{i\pi/6} = \frac{1}{2}\sqrt{3} + \frac{1}{2}i$

   Use this fact to compute the following: Let $v$ be the vector $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Rotate this vector by an angle $\pi/6$ (counter clockwise). What is the result?

15. If $z$ is a non-zero complex number, and if $\mathfrak{p}$ happens to be equal to $z^{-1}$, then the absolute value of $z$ must be:

16. Compute the quaternion $(i + j - k) \cdot (1 - i - j)$.