Second try at Test 1, Feb 22 2005, MAS3301

1. Use the Euclidean algorithm to find two integers \( s, t \) for which 
\[ 101s + 150t = 1. \]

2. Find the two complex solutions \( a \pm bi \) of the equation \( x^2 + 4x + 8 = 0 \).

3. Write \( \sqrt{3} - i \) in polar coordinates: \( \sqrt{3} - i = re^{i\alpha} \) where the real numbers \( r, \alpha \) are:

4. Which field axiom(s) is/are not satisfied by the set \( \{0, 1, -1\} \)?

5. Compute all complex number(s) \( z \) for which \( \overline{z} + 3z = 4 + 4i \).

6. Find a polynomial with integer coefficients that has 
\( 3^{2/3} - 3^{1/3} \) as a root.

7. Let \( z \) be the quaternion \( 1 + i + j \).
   (a) What is the absolute value of \( z \)?
   (b) What is the conjugate of \( z \)?
   (c) Compute \( z^{-1} \), the multiplicative inverse of \( z \).
   (d) Explain why \( zuz^{-1} \) has the same absolute value as \( u \) for any quaternion \( u \).

8. Let \( u = 3 + i + j \) and \( v = 3 + 3i + j \).
   (a) Compute the norm of \( u \) and the norm of \( v \) (hint: the norm of \( u \) 
is the square of the absolute value of \( u \)).
   (b) Compute \( uv \).
   (c) Write 209 as a sum of four squares.