Second try at Test 1, Feb 22 2005, MAS3301

1. Use the Euclidean algorithm to find two integers s, t for which $101s + 150t = 1$.

2. Find the two complex solutions $a \pm bi$ of the equation $x^2 + 4x + 8 = 0$.

3. Write $\sqrt{3} - i$ in polar coordinates: $\sqrt{3} - i = re^{i\alpha}$ where the real numbers r, α are:

4. Which field axiom(s) is/are not satisfied by the set $\{0, 1, -1\}$?

5. Compute all complex number(s) z for which $\overline{z} + 3z = 4 + 4i$.

6. Find a polynomial with integer coefficients that has $3^{2/3} - 3^{1/3}$ as a root.

7. Let z be the quaternion $1 + i + j$.

   (a) What is the absolute value of z?

   (b) What is the conjugate of z?

   (c) Compute z^{-1}, the multiplicative inverse of z.

   (d) Explain why zuz^{-1} has the same absolute value as u for any quaternion u.

8. Let $u = 3 + i + j$ and $v = 3 + 3i + j$.

   (a) Compute the norm of u and the norm of v (hint: the norm of u is the square of the absolute value of u).

   (b) Compute uv.

   (c) Write 209 as a sum of four squares.