Turn in on Feb 20: Two proofs for the first part of Ex 5 in Section 2.2 (the second part about transcendental numbers, you may omit that).

Hints:

1. Recall from the Hotel Infinity handout that if you have a hotel with rooms $\{1, 2, 3, \ldots\}$ and you have current guests $\{c_1, c_2, c_3, \ldots\}$ and new guests $\{n_1, n_2, n_3, \ldots\}$ then we can fit all current and all new guests in rooms $\{1, 2, 3, \ldots\}$.
   In terms of set theory: there is a bijection from $\{c_1, c_2, \ldots, n_1, n_2, \ldots\}$ to $\{1, 2, 3, \ldots\}$. That bijection was given in the Hotel Infinity handout. Specifically, current guest c_k was sent to room $2k$. Then new guest n_k was sent to room $2k-1$. Last week I used the same idea to show that:

   Lemma 1: if A is an infinite set, and B is a countable set, then there is a bijection from $A \bigcup B$ to A.

   Now $\mathbb{Q}$ is the set of rational numbers, $\mathbb{R}$ is the set of real numbers, and $\mathbb{R} - \mathbb{Q}$ is the set of irrational real numbers. Notice that $\mathbb{R} - \mathbb{Q}$ is an infinite set. Now use Lemma 1 to prove that $\mathbb{R} - \mathbb{Q}$ has the same cardinality as $\mathbb{R}$.
   (You do not need to prove Lemma 1, instead, you may use that Lemma 1 is true **for any** infinite set A and **any** countable set B).

2. Read item 14 in handout "List of facts for Chapter 2". I gave an example in class today that you can use this to prove things that would otherwise be hard to prove (find that example in your notes as it may give you a hint).

   Now let $A = (0, \infty)$ and consider the function $f : A \to \mathbb{R} - \mathbb{Q}$ which does the following: If $x \in \mathbb{Q}$ then $f(x) = -\sqrt{2} \cdot x$. If $x \notin \mathbb{Q}$ then $f(x) = x$. Briefly explain why this f is an injective function from A to $\mathbb{R} - \mathbb{Q}$. Then use that to explain (very similar to what we did in class today, using item 14 from "list of facts") that $\mathbb{R} - \mathbb{Q}$ must have the same cardinality as $\mathbb{R}$.