# Hotel Infinity (Harm Derksen's version of a story by David Hilbert)

Once upon a time, there was a hotel called *Holiday Infinity* with an infinite number of rooms, numbered $1, 2, 3, \ldots$
One day, when all rooms were already occupied, a new guest arrived and asked for a room. The receptionist came up with a clever solution: All current guests were asked to move up 1 room. So guests in room $n$ moved to room $n + 1$ like this:

$$1 \mapsto 2 \mapsto 3 \mapsto 4 \mapsto \cdots$$

This freed up room number 1 which was given to the new guest.

The next day, *Comfort Infinity*, the hotel next door, had a fire. It too had infinitely many rooms. All of its infinitely many guests survived but they all needed a room in *Holiday Infinity*. Once again, the receptionist of *Holiday Infinity* found a clever solution: Guests in room $n$ were asked to move to room $2n$ like this:

$$1 \mapsto 2, \quad 2 \mapsto 4, \quad 3 \mapsto 6, \quad 4 \mapsto 8, \quad \cdots$$

This freed up the rooms with numbers $1, 3, 5, 7, \ldots$ in *Holiday Infinity*, enough to house all guests from the now burnt *Comfort Infinity*.

Once people started to realize that one Infinite Hotel can hold all guests from two Infinite Hotels, a price war arose. This led to the sudden bankruptcy of a hotel chain called *Days Infinity*. This hotel chain had infinitely many hotels $(1, 2, 3, \ldots)$ each of which had infinitely many rooms $(1, 2, 3, \ldots)$.

After the bankruptcy, all guests of *Days Infinity* hotel chain suddenly needed a room in *Holiday Infinity*. Unfortunately, the receptionist quit in despair. Nevertheless, there would have been a solution! Can you find it? (use Theorem 1 on page 22 in the book).