Handout OP: Organizing Proofs.

In math so far, you computed with numbers and functions. In this class you ***compute with statements***. Organization is key to prevent computation errors. In every statement S that occurs in your proof, it must always be clear if S is:

- Given
- To Prove (TP)
- Assumed
- Derived

Rules:

1. Never assume the TP statement (do not assume the conclusion).

2. Never prove Given or Assumed statements (the premises).

3. For any statement S that occurs anywhere in your proof, it must be clearly indicated if it is Given, TP, Assumed, or Derived.

4. To indicate that S is assumed, use the key word "Assume".

5. Derived statements are all statements that you managed to prove from Given + Assumed statements. If S is a derived statement, you must indicate that with key word(s). There are many key words you can choose from: hence, thus, therefore, so, because, it follows that, Then (as first word in a sentence, not as part of an if-then), etc.

6. Do not write $A \implies B$ to indicate that B is derived from A.
(*A* hence B means both are true, but $A \implies B$ does not! See truth tables).

7. It is OK if an Assumed or Derived statement is false! If you correctly derived a false statement like $p \wedge \neg p$ then you proved that at least one of the assumptions is false. That can be useful, see WP#3,4,7,8.

8. Do not drop quantifiers! (regardless of whether they are written with symbols $\forall$ and $\exists$ or with key words like "for all" and "for some").
Suppose for example TP: $\forall_{x \in A} \exists_{y \in B}\, P(x, y)$.
A direct proof starts with: "Let $x \in A$".
Errors on tests are often caused by a dropped quantifier. To minimize the chance of that, after "Let $x \in A$" ***write this***: TP: $\exists_{y \in B}\, P(x, y)$
and then check which of WP#3 or WP#6 looks the most promising.

9. These three statements differ, do not mix them up:
(1) $P(x)$ for all x in A. (same as $\forall_{x \in A}\ P(x)$)
(2) $P(x)$ for some x in A. (same as $\exists_{x \in A}\ P(x)$)
(3) $P(x)$ for no x in A. (same as $\forall_{x \in A} \neg P(x)$)

10. Do not use undeclared variables (to avoid mixups like in items 8 and 9).

11. To define x (e.g. WP#6) don't write $\ldots = x$. Instead write: Let $x := \ldots$

12. It helps a lot if you give each statement a separate line and label. If you use a given/assumed/derived statement that is not on the preceeding line, then cite its label. Example: Then p by (1).
That tells the reader that p follows from the preceeding line and line (1). But it helps you too! (if you are stuck, look at given/assumed statements you have not yet used).

13. Use official formal definitions, not your interpretation of the definitions!