1. Simplify the following cardinal numbers to the point where they look like 0, 1, 2, . . ., or \( \aleph_0 \), or \( c \), or \( 2^c \), or \( 2(2^c) \), etc.

\[
\begin{align*}
\aleph_0 + \aleph_0 & \quad 2^{\aleph_0} + 2^{\aleph_0} \\
o(\{3, 3, 3\}) & \quad (2^c)^2 \\
o(P(\emptyset)) & \quad o(P(\mathbb{Q}) - P(\mathbb{N})) \\
o(P(\mathbb{R})\mathbb{R}) & \quad o(\emptyset^\mathbb{R} \times \mathbb{R}) \\
o(\mathbb{R} \times \mathbb{N}) & \quad o(\mathbb{R}^\mathbb{N}) \\
7^{\aleph_0} & \quad o(P(\mathbb{N})) \\
\end{align*}
\]

2. Let \( A, B, C \) be sets.

(a) Write down the definition of \( o(A) \leq o(B) \).

(b) We know that if \( o(A) \leq o(B) \) and \( o(B) \leq o(C) \) then \( o(A) \leq o(C) \).

Write down a proof for this rule using only the definition of \( \leq \).

3. (a) Give the definition of cardinal addition: if \( D \cap E = \emptyset \) and \( d = o(D) \) and \( e = o(E) \) then \( d + e \) is defined as:

(b) Let \( A \) be an infinite set, let \( B \) be a subset and let \( C = A - B \).

Suppose that there is a bijection from \( B \) to \( C \).

Prove that there is a bijection from \( A \) to \( C \).

(c) Give the definition of cardinal multiplication.

(d) Let \( A, B, C \) as in part (b), and assume that \( B \) and \( C \) are not empty.

Prove that there is a bijection from \( A \) to \( B \times C \).

4. Let \( \mathbb{N}^* = \{1, 2, 3, 4, \ldots\}, E = \{2, 4, 6, 8, \ldots\}, D = \{1, 3, 5, 7, \ldots\} \).

So \( E = \{\text{all even positive integers}\} \), and \( D = \{\text{all odd positive integers}\} \).

(a) Give a bijection \( f : \mathbb{N}^* \to E \) (write down: \( f(n) = \ldots \))

(b) Give a bijection \( g : \mathbb{N}^* \to D \).

(c) Explain why parts (a), (b), and Ex.3(a), show: \( \aleph_0 + \aleph_0 = \aleph_0 \).
5. Take home: Suppose that for each $n \in \mathbb{N}^*$ you are given a subset $A_n \subseteq \mathbb{R}$. Suppose that $\mathbb{R} = A_1 \cup A_2 \cup A_3 \cup \cdots$. Show that at least one of those sets $A_n$ must be uncountable.

6. Take home: Let $A = \{ S \subseteq \mathbb{R} \mid S \text{ countable}\}$. So $A$ is the set of all countable subsets of $\mathbb{R}$, in other words: $S \in A$ if and only if $S$ is countable and $S \subseteq \mathbb{R}$.

Prove that $o(A) = c$.

Hint: Find an onto function $F : \mathbb{R}^{\mathbb{N}^*} \rightarrow B$ where $B = A - \{\emptyset\}$.