Rules:

- Homework must be turned in at the beginning of class on the due date. Work turned in within 2 days after this deadline will be considered late, and credit will be reduced by 25%. Work turned in any later will be given a grade of 0.

- Write NEATLY and show your work. If I can’t read it, I won’t grade it. Correct answers are insufficient unless supported by correct reasoning.

- Do your own work. You can discuss the problems with others in the class (or me), but the assignment you turn in must be your own work. If I feel that your assignment has been copied from someone else, both assignments will receive a grade of 0.

- Label all figures, whether hand-drawn or computer-generated. Clarity of presentation is ESSENTIAL.

1. Solve the following difference equations subject to the specified initial $x$ values, and sketch the solutions.

   (i) $x_n - 5x_{n-1} + 6x_{n-2} = 0; \quad x_0 = 2, \quad x_1 = 5.$

   (ii) $x_{n+1} - 5x_n + 4x_{n-1} = 0; \quad x_1 = 9, \quad x_2 = 33.$

2. Convert the following system of difference equations to single higher-order equations and find their solutions. Indicate whether the solution increases or decreases, and whether oscillations occur.

   (i) $x_{n+1} = x_n + y_n; \quad y_{n+1} = 2x_n.$

   (ii) $x_{n+1} = -x_n + 3y_n; \quad 3y_{n+1} = y_n.$

3. (a) Solve the first order equation: $x_{n+1} = ax_n + b$, where $a, b$ are constants.

   (b) Consider the difference equation: $aX_{n+2} + bX_{n+1} + cX_n = d$. If $d \neq 0$, then this equation is called nonhomogeneous. Show that $X_n = K$ solves this equation for some constant $K$ that depends on the model parameters $a, b, c, d$, and as long as $a + b + c \neq 0.$
This is called a particular solution.

(c) Suppose the solution to the corresponding homogeneous equation \( aX_{n+2} + bX_{n+1} + cX_n = 0 \) is \( X_n = c_1 \lambda_1^n + c_2 \lambda_2^n \) for arbitrary constants \( c_1 \) and \( c_2 \) and where \( \lambda_{1,2} \) are eigenvalues of this problem. This is called a complementary solution. Show that the general solution: \( X_n = c_1 \lambda_1^n + c_2 \lambda_2^n + k \) will be a solution to the nonhomogeneous problem in (b).

4. Convert the following system to matrix-vector form, and define the vectors and matrix.
\[
\begin{align*}
3x + 4y &= 2 \\
-x + 3y &= 0
\end{align*}
\]

5. Consider the second order difference equation \( x_{n+2} + bx_{n+1} + cx_n = 0 \), with complex eigenvalues \( \lambda_{1,2} = a \pm ib \). Show that \( r^n \cos(n\theta) \) and \( r^n \sin(n\theta) \) are themselves solutions to the difference equation. Here, \( r = \sqrt{a^2 + b^2} \) and \( \theta = \arctan(b/a) \).

Hint: the real part of a complex number \( z \) equals \((z + \overline{z})/2\) and the complex part equals \(i(\overline{z} - z)/2\).

6. Find the eigenvalues and eigenvectors of the following matrices:
\[
A = \begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}
\]

7. Find the general solutions to each of the following difference equations, and then find the particular solution satisfying \( x_0 = -1 \) and \( y_0 = 1 \). (Hint: problem 6 may be helpful):
\[
\begin{align*}
\text{(i)} \quad x_{n+1} &= 3x_n + 4y_n \\
y_{n+1} &= 4x_n - 3y_n \\
\text{(ii)} \quad x_{n+1} &= 2x_n \\
y_{n+1} &= -x_n + 3y_n
\end{align*}
\]

8. Find the general solution to the following difference equation, and then find the particular solution satisfying \( x_0 = 2 \) and \( x_1 = 3 \). (Hint: problem 3 may be helpful):
\[
x_n - 5x_{n-1} + 6x_{n-2} = 4
\]

9. Find the general solution to the following difference equation (Hint: complex eigenvalues): \( x_{n+1} + x_n + x_{n-1} = 0 \).
10. Use any computing software (such as Excel) to graph solutions to the Annual Plant Propagation model for the following sets of parameter values: $\alpha = 0.6, \gamma = 2.0, \sigma = 0.8, \beta = 0.3, P_0 = 100, P_1 = 109.97$ and $\alpha = 0.5, \gamma = 2.0, \sigma = 0.8, \beta = 0.25, P_0 = 100, P_1 = 96.57$. Play around with the values of $\alpha$ and $\beta$, and observe how the qualitative behavior of solutions changes.

$$p_{n+1} = \alpha \sigma \gamma p_n + \beta \sigma^2 (1 - \alpha) \gamma p_{n-1}$$