A Systems Model of the Effects of Training on Physical Performance

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Abstract—A systems model is proposed to relate a profile of athletic performance to a profile of training. The general model assumes that performance has four components: endurance, strength, skill, and psychological factors. Each of these factors is discussed and ascribed a transfer function. A major problem is the quantification of both training and performance. The case of a swimmer is studied in detail. It is shown that if a time series of training impulses is used as input, his performance in 100 m criterion performances can be modeled rather well with a simple linear system. The major conclusion is that performance appears to be related to the difference between fitness and fatigue functions. The fitness function is related to training by a first-order system with time constant 50 days, whereas the fatigue function is related to training by a similar system with time constant 15 days. An appendix is provided to show how these systems can be simulated on a simple electronic calculator. The relationship of these relatively short-term effects on the individual performer (six months) to longer term effects on the same individual is also discussed.

Introduction

IN SPITE of the many applications of systems models to physiology, few attempts have been made to model quantitatively the effects of physical training on human athletic performance [1]. Although an effort has been made in athletics to provide comparative ratings of optimal performances in running events over different distances and extend these concepts to training schedules and specifications [2], [3] the method has offered no conceptualization of the training process itself. A question which has particular significance is: how does training modify performance throughout the whole training period? The intimate details of performance growth are usually never revealed since no real criterion performances are attempted in noncompetitive or build-up periods. The training process is thus obscured in the midst of the most arduous preparation. Intuition or experience on the part of an athlete or coach determines gradual modification of the degree of intensity and/or duration of training necessary to produce an optimal performance at a particular time in the future. If the training-dependent profile of optimal performance may be modeled, however, and particularly if its nuances may be directly physiologically, psychologically, or nutritionally related, a greater understanding of preparation for optimal performance will be achieved. The essence of the procedure, therefore, is to be able, during an extended period, to model the athlete's responsiveness to training both during the time when the latter is arduous and debilitating and also during "tapering" when the athlete is easing training and rebounding, hopefully to his best ever performances. If the profile is followed through several cycles, the model be-

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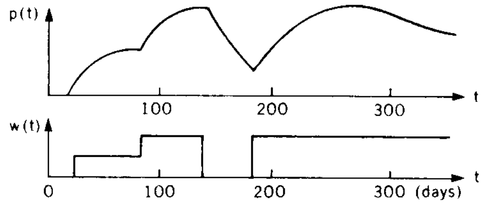

Fig. 1. General relationship between physical training $w(t)$ (system input) and physical performance $p(t)$ (system output).

comes predictive in the sense that greater and greater precision is achieved in matching training-derived criterion performances to actual performances, and the attainment of optimal performance through proper tapering and avoidance of over-training can be controlled. In this way also the model can ensure that the best performance is achieved at a precise point in time when it is most desirable, i.e., during an important competition and not in a training session.

In modeling living systems, two pure strategies are available. In the first, referred to as an *analytic strategy*, a system is broken down into its constituent components, and each is described by applying the laws of physics and chemistry. An example is the well-known equations which govern the differences in ion concentrates across a cell membrane. The second, contrastingly referred to as the *black-box strategy*, makes no assumptions about the constituents of the system and only considers input–output relationships. In this case the output which results from a known input is used to obtain a general *transfer function* for the system. Generally, neither of these pure strategies is sufficient for nontrivial systems, and a mixed strategy is used. The respiratory system models of Grodins [4] and Milhorn [5] or the temperature control model of Stolwijk [6] are examples of useful systems models which rely on both strategies. In the present approach to modeling the effects of training we must rely largely on input–output data (much of it difficult to quantify) with the system initially regarded as a "black box." A long-term aim is to identify successively all the components of the system and to describe their function. In this way, improved models and relevant accompanying experiments may lead to a better understanding of the various component mechanisms involved in exercise [7], [8].

An athletic coach or exercise physiologist, if asked to describe the effects of training on performance, will probably agree that they are generally as described in Fig. 1, where a sudden moderate increase in training $w(t)$ causes performance $p^{(t)}$ to rise to a limit with a time constant (τ) of 30–50 days. A further increase in training causes a further rise; in each case this rise would be proportional to the difference between the potential maximal performance inherently determined and the presently attained performance $p(t)$ level. After the cessation of continuous training, performance will decay exponentially back to a lower level. Thus it appears that the system can be described grossly by a simple first-order differential equation:

$$\frac{dp(t)}{dt} + \frac{1}{\tau}\,p(t) = w(t). \qquad (1)$$

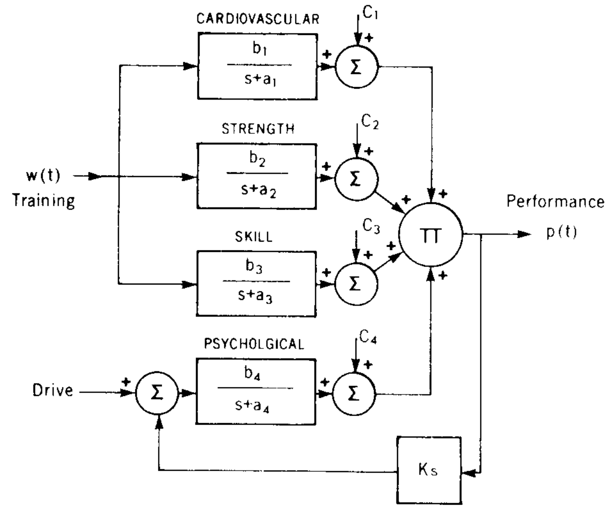

Fig. 2. Multicomponent model to explain effects of different forms of training on performance. Parameters are specific to individual performer, type of training, and type of performance.

The corresponding transfer function is

$$G(s) = \frac{P(s)}{W(s)} = \frac{1}{s + 1/\tau} \qquad (2)$$

and in terms of convolution:

$$\begin{aligned} p(t) &= w(t) * g(t) \\ &= w(t) * e^{-t/\tau} \end{aligned} \qquad (3)$$

where $*$ indicates convolution, $g(t)$ is the impulse response $e^{-t/\tau}$, and τ is the time constant (30–50 days). Any statements about the units of $p(t)$ and $w(t)$ are deliberately omitted. Although this elementary description can lead to a greater understanding of the system, it is far too simple. For example, if training is continued at a high level for a long period of time (Fig. 1), then at about 150 days from commencement of training performance may begin to fall. This is known as over-training. Furthermore, in many cases the dynamic response to step changes is not a simple exponential.

Components of Human Performance

Fig. 2 shows one proposed systems model which contains elements representing what are originally conceived as strong determinants of performance. These elements are 1) endurance, 2) strength, 3) skill, and 4) psychological factors. Different tasks involve each of these components to a greater or lesser extent. For example, the performance of a marathon runner is dominated by endurance, the javelin thrower by skill, and the weight lifter by strength. However, all human performance has a psychological component, particularly if it must be maintained for long periods in arduous conditions or repeated frequently as in training.

Endurance

This component involves the respiratory, cardiovascular, and cell metabolism systems in the body. If the muscle cells are to perform sustained work they must use aerobic processes, i.e., oxygen must be used to utilize the energy stored in glycogen. If sufficient substrate is available, a condition determined by the current nutritional state of the athlete, then the level of performance will be limited by the rate at which oxygen can be supplied to the cells. One aim of endurance training is to increase this oxygen supply. The oxygen supply may be limited by the lungs, the cardiac output, the number of red blood cells, and by the peripheral circulation of the blood to the muscles [9]. While all of these factors may be modified by exercise training, it has been suggested that in the healthy adult, performance will normally be limited by the peripheral circulatory system which delivers blood to the muscle [10]–[13].

Endurance training can be conducted under carefully controlled conditions. The subject's training and performance can be measured accurately on a bicycle ergometer (for example) and other components (strength, skill, psychological) can have minimal effect on the results. When work cannot be measured directly (as in walking, running, swimming, etc.) indirect measures must be used. Heart rate (in beats per minute) is often used as an indicator of performance—the PWC_{170} test, for example, is based on this (Astrand [14]). Certainly, if the stroke-volume of the heart is maximal, the heart rate will be a measure of cardiac output and hence of the oxygen delivered to the tissue. Heart rate is probably a useful indicator for the average individual but less reliable for the highly trained athlete (e.g., a marathon runner). A much more reliable indicator of work is oxygen uptake, but this is difficult to measure directly and continuously during training in the field. Additionally, the variable nature of the speed and rest pauses encountered in training makes the measurement of oxygen uptake even more difficult.

Many experiments indicate that endurance training has an effect on performance similar to that illustrated in Fig. 1 and quantified in (1)–(3). The time constant seems to be about 30–50 days. The results are complicated by the fact that heavy endurance training causes fatigue, and thus performance will be lower in the first few days after a training session. This is discussed more fully below. Banister [15] has shown that in the untrained individual, a continuous training session is more effective than one in which the work levels are alternately high and low, even if the average work is the same as in the continuous session. Probably a training session should last at least 20 minutes if it is to be effective. Several investigators have remarked on the relative contributions of intensity, duration, and frequency of training for optimal effects [16]–[18].

Strength

A subject on a bicycle ergometer with weak leg muscles might have his performance limited by strength rather than aerobic capacity. Clearly for a weightlifter, strength is more important than endurance. Strength can be increased by functional hypertrophy of the muscles and by improving the neural organization for recruiting muscle fibers. Functional hypertrophy is the result of repeated use of the muscles at maximal or close to maximal levels. The maximum force which can be exerted by the muscle will then also increase in a manner similar to that described in Fig. 1 and (1)–(3). The time constant in this case seems to be 20–40 days. With disuse, strength will decay. The force developed by a muscle in a particular movement can also be increased somewhat by adaptation of the neural organization which recruits the muscle fibers. This might be thought of as a form of motor learning. In contrast to functional hypertrophy, this effect may have a time constant of perhaps seven days and requires only that the movement be repeated a number of times each day.

It might appear that the input and output of the strength component of performance are easy to measure and quantify. Certainly, training can be quantified as the number of times a muscle group contracts to a given force, and performance can be quantified as the maximum force which can be developed. Unfortunately, almost all human activity involves strength training to some extent, and endurance training of swimmers (for example) will certainly develop their muscles. Thus for many types of activity it is almost impossible to isolate quantitatively the strength training—fortunately, for many activities, strength will not limit performance.

Skill

This component is very important in some activities (e.g., javelin throwing, high jump) and relatively unimportant in others (e.g., exercise therapy, long-distance running). The acquisition of skills is a form of motor learning and has been studied extensively [20]. Learning curves for some activities are similar to Fig. 1, but others show a jump phenomenon. The jump in performance occurs when the performer suddenly masters some action. Fortunately, these sudden jumps in performance are easily identified since almost all other changes occur relatively slowly.

Psychological Factors

All human performance of the type considered in this paper is voluntary and is carried out because of some "drive." Many of the activities may cause discomfort or pain, and if the subject is not strongly motivated, it is unlikely that he will perform maximally. In particular, the drive to perform may depend on the performance, and thus feedback exists from output to input. One possible explanation for "overtraining" (see Fig. 1) is that after a prolonged period of training, interest is lost and drive drops. This results in a fall in performance which itself causes a further decrease in drive so that performance falls even more and so on. This is a difficult component to quantify since psychological measurements are unreliable, and there are wide differences between individuals [19].

A Multicomponent Model

A speculative first attempt to combine the components of performance into one model is shown in Fig. 2. The constants will vary greatly for different activities, but the structure can explain most of what we know about training and performance. The training input $w(t)$ will have an effect on the endurance, strength, and skill components determined by their individual transfer functions.

Before any model can be meaningful, however, it is necessary to define input and output in quantitative terms. In the case of athletic activity, the training input becomes remarkably complex and is really multidimensional. It includes the general components shown in Fig. 2: endurance, strength, and skill training, and psychological drive. Also, the parameters of the model of the performer are not really constant and dependent on such pseudo-inputs as inherent ability and current nutritional state. The model postulates a feedback to the psychological component dependent on the rate of change of performance. Additional feedback loops directly from skill and strength to the psychological box may be postulated. In addition, there are possible feed-forward loops from the psychological component to skill, strength, and cardiovascular components, attributes which may be given such terms as "concentration," "motivation," and "biofeedback information," respectively. All of these loops might alter the transfer functions of skill, strength, and cardiovascular function before they aggregate in performance. Thus the model of Fig. 2 is a mere skeleton of what a complete model may eventually be.

Given the general character of only the four components of human performance described in detail above and shown in Fig. 2, it seems that the most informed guess we can make about transfer functions representing the four variables and their aggregation is that they are of the form:

$$G(s) = \frac{a}{s + b} \tag{4}$$

where $1/b$ is the time constant and a/b is the "gain." The way in which the components should be combined, however, is by no means clear. Some components may have additive effects while others will be multiplicative. This is handled without loss of generality by adding a constant to each component and mixing them in a multiplier; i.e., for two components x_1 and x_2, the mixing gives

$$(x_1 + c_1)(x_2 + c_2) = x_1x_2 + x_1c_2 + x_2c_1 + c_1c_2,$$

so that there is both multiplicative and additive interaction. Most simply, the psychological component depends not on training but on both input "drive" and a positive feedback dependent on the rate of change of performance with the possibility already mentioned, but not modeled here, of feed-forward loops from the psychological component to skill, strength, and cardiovascular components, respectively.

Although this general model has obvious deficiencies and many variations could be proposed, it accounts for the major determinants of performance. In any event, there is as yet no multicomponent activity which has been studied in sufficient detail to test the details or the framework. Each component transfer function should probably be nonlinear and should definitely contain a saturation type limiter, since there are physiological limits to how much training can be tolerated (which themselves change as training progresses). In spite of this, the framework should be useful in the design of crucial experiments in the future. In the simplest, most imperfect case, the quantity of training itself may be taken as the input and the time of a criterion performance the sole output. As better quantification of other components of the model is made, they may be incorporated until a sophisticated model is achieved. The simplest input–output relationship between the quantity of training and time of a criterion performance produces surprising insight into and allows conceptualization of the training process. It is the model of this process which is of prime interest in this paper.

A Model of a Swimmer

Although the method is being used to model a variety of athletic performances including running, cycling, and swimming, the most comprehensive data in training and criterion, performances throughout a training regimen have been collected on a swimmer, and these data are used to illustrate the model below. Only training and performance levels are used to follow the training process in this initial modeling procedure. Although it is considered that the psychological component undoubtedly plays a role, no attempt was made to account for it here, and only minimal skill or strength components are involved at this swimmer's high stage of development.

Quantification of Training

Training programs for competition swimmers are complicated and difficult to quantify. The subject in this study underwent a regular carefully-supervised program of flexibility exercises, weight exercise (to develop strength), and swimming. The swimming comprised a warm-up (typically 500 m), low-quality activity (long distances swum relatively slowly, e.g., 3000–5000 m in each session) and high-quality activity (short distances swum quickly with long rests between repeats, e.g., 100 m at intervals of three or four minutes). Without very careful physiological measurements which would be unrealistic to repeat routinely at every training session, it is difficult to compare the effects of swimming different distances with different strokes at different speeds. Thus an arbitrary categorization has been adopted taking into consideration the "feel" of the swimmer himself for the training; this appears reasonable but is certainly open to improvement.

Different levels of swimming are assigned arbitrary intensity factors which are used to calculate arbitrary training units (ATU).

1) Warm-up: Intensity = 1, each 100 m swum ≡ 1 ATU
2) Low quality: Intensity = 2, each 100 m swum ≡ 2 ATU
3) High quality: Intensity = 3, each 100 m swum ≡ 3 ATU.

Weight training is nominally designed to improve the strength of the swimmer's muscles and as such should have contributed to a separate component of performance as suggested in Fig. 2. However, the swimmer, himself a trained exercise physiologist, reported that a large number of pulls (about 500 in each session) against fairly light loads

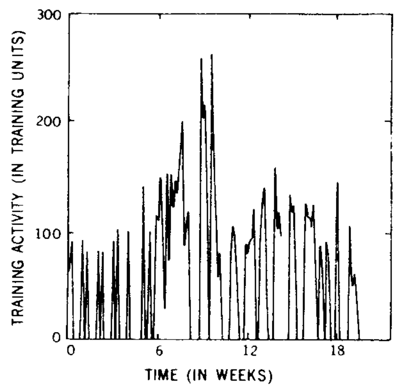

Fig. 3. Aggregate training activity of swimmer in 1970–1971 season. Training is measured in arbitrary training units (see text).

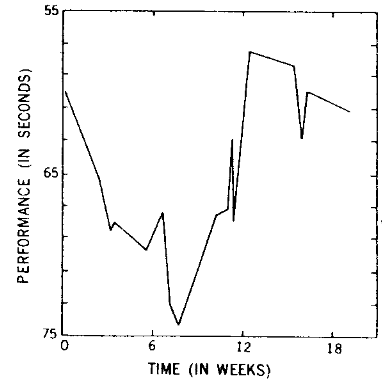

Fig. 4. Performance of swimmer on 100-m time trials in 1970–1971 season.

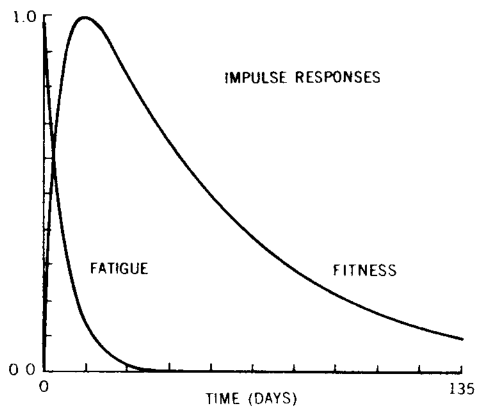

Fig. 5. Impulse responses used for fitness and fatigue functions.

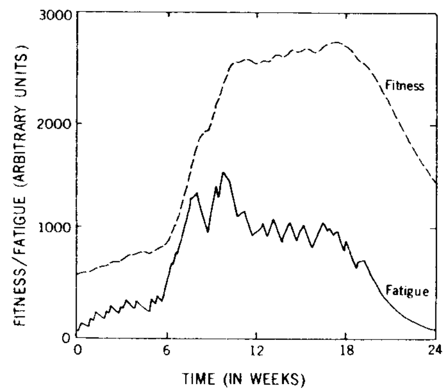

Fig. 6. Profiles of fitness and fatigue functions in 1970–1971 season. Units are arbitrary and depend on arbitrary training units.

probably had minimal effect on his strength but was roughly equivalent to exercise in high-quality swimming. Consequently, a weight session of 500 pulls has been equated to a high quality swim of 1000 m or to 30 ATU (i.e., 1000 × intensity factor 3 = 3000 ÷ 100 = 30 ATU), and these values were added to swim training ATU's to get a combined ATU for each training session.

On a typical day there might be 500 pulls in the weight session, a 500-m warm-up swim, an 8000-m low-quality swim and a 500-m high-quality swim to give a training impulse for that day of 30 + (1 × 5) + (2 × 80) + (3 × 5) = 210 ATU. Fig. 3 shows the profile of training impulses in aggregate ATU's for each day throughout training.

The criterion performance of the swimmer is taken to be his time to swim 100-m time trials. The actual times for the 1970–1971 season are plotted in Fig. 4. For clarity, the shortest times (i.e., best performance) are plotted positively.

To test the "conventional wisdom" embodied in the notions illustrated in Figs. 1 and 2 and (1)–(3), training $w(t)$ was used as input to the transfer function $G(s)$ with $\tau = 50$ days. Since the training session is quite short (a few hours) compared to the time constants of the system (10's of days) it is legitimate to regard it as an impulse, the area of which is measured in our arbitrary training units. Thus (3) implies that each impulse of training will contribute an impulse response of the form $e^{-t/\tau}$ to the modeled performance immediately after each training session. This is considered to be nonphysiological since the systems involved require many hours to adapt in response to training. Consequently, the impulse response was changed to

$$g_2(t) = (e^{-t/\tau_1} - e^{-t/\tau_2}) \tag{5}$$

with $\tau_1 = 50$ days and $\tau_2 = 5$ days. This impulse response is illustrated in the curve labeled "Fitness" in Fig. 5.

The model performance $p(t)$ obtained by convolving the training input $w(t)$ with the system impulse response $g_2(t)$ is shown in Fig. 6 by the curve labeled "Fitness." While this result for performance may seem intuitively reasonable and desirable from the point of view of a coach, it certainly is not like the actual performance in Fig. 4. Here in fact, after repeatedly swimming up to 12 000 meters per day for several weeks (with total training up to 280 ATU per day) the swimmer suffers serious fatigue and just cannot perform very well. Thus it is reasonable to hypothesize, and this is the very essence of the conceptualization which the model has produced, that training introduces a time function of fatigue $f(t)$ with a fatigue impulse response $h(t)$ (shown in Fig. 5 as the "fatigue" curve) such that

$$f(t) = h(t) * w(t) \tag{6}$$

$$= e^{-t/\tau_3} * w(t) \tag{7}$$

where τ_3 = fatigue time constant = 15 days (guessed first and refined by iteration). The main feature of the fatigue impulse response is its relatively short time constant. The predicted fatigue $f(t)$ is shown labeled in Fig. 6. The rationale then is that

$$\text{model performance} = \begin{pmatrix}\text{fitness from}\\ \text{training model}\end{pmatrix} - K\begin{pmatrix}\text{fatigue from}\\ \text{training model}\end{pmatrix}$$

or

$$a_1(t) = p(t) - Kf(t) \quad (8)$$

$$= g_2(t) * w(t) - Kh(t) * w(t)$$

$$= [g_2(t) - Kh(t)] * w(t)$$

$$= [(e^{-t/\tau_1} - e^{-t/\tau_2}) - Ke^{-t/\tau_3}] * w(t). \quad (9)$$

The equivalent block diagram and transfer functions are shown in Fig. 7.[1] When this was implemented with $\tau_1 = 50$ days, $\tau_2 = 5$ days, $\tau_3 = 15$ days, and $K = 2.0$, the profile for $a_1(t)$ indicated by the continuous line in Fig. 8 was obtained. Since $a_1(t)$ is in arbitrary units (dependent upon the arbitrary training units defined above) linear regression is used to fit the profile to the actual performance (shown in Fig. 4). Then modeled performance $a_2(t)$ in seconds is given by

$$a_2(t) = 66.5 - 0.0075 \times a_1(t)$$

$$= 66.5 - (0.0075p(t) - 0.015f(t)). \quad (10)$$

This is directly compared with the actual performance in Fig. 8. Iterative and indeed interactive modeling at the computer graphics terminal adjusted τ_1, τ_2, τ_3, and K to produce the closest model of the criterion performances from those created by the training fatigue model [22]. While $a_2(t)$ follows the general profile of the actual performance, there are wide discrepancies for many of the points. Although some of these variations may be ascribed to a simplified model or to arbitrary quantification of the training, it is believed that the major difficulties are the uncontrolled and unrecorded factors in the subject's life which affected his time trials. For example, as a first year university student striving to maintain a high grade point average, he frequently worked late on assignments and occasionally attended parties the night before the trial.

This contention is borne out by the data available for the same swimmer in his fourth season at university (1973–1974). The level and profile of the training $w(t)$ is quite similar to that for the 1970–1971 season as shown in Fig. 9. However, although the model and actual performances in Fig. 10 have the same profile as for the 1970–1971 season, the change in performance during the season is reduced by

[1] Initial conditions for these differential equations are unknown since detailed records of training for the previous season are not available. The initial conditions for $p(t)$ and $f(t)$ were estimated by finding the values at the end of the training season under study (i.e., $p(135)$, $f(135)$). These are largely independent of the initial conditions, since the largest time constant is 50 days. The initial conditions were then approximated by assuming $p(135)$ and $f(135)$ were the values at the end of the previous season and allowing them to decay for the period of inactivity between seasons (75 days).

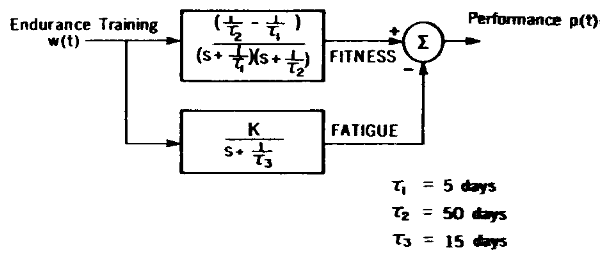

Fig. 7. Block diagram and transfer functions for model of swimmer.

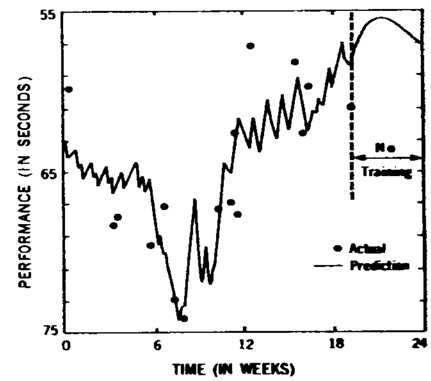

Fig. 8. Model performance matched by linear regression to actual performance in 1970–1971 season.

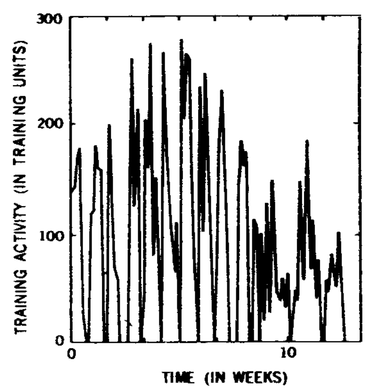

Fig. 9. Aggregate training activity in 1973–1974 season.

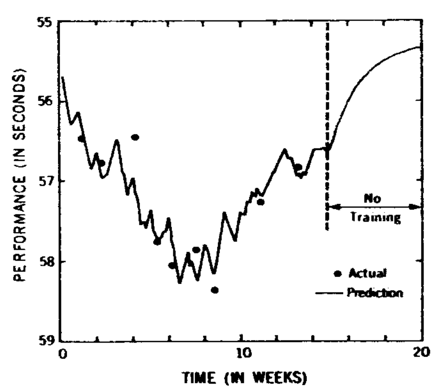

Fig. 10. Model performance matched by linear regression to actual performance in 1973–1974 season.

an order of magnitude (i.e., 1.9 s in 1973–1974 compared to 17 s in 1970–1971). In spite of this rather major change in the range of performance, (9) models performance very well. In this case $\tau_1 = 50$, $\tau_2 = 5$, $\tau_3 = 15$, $K = 10.0$, and

$$a_2(t) = 55.5 - 0.00017 \times a_1(t)$$
$$= 55.5 - (0.0017p(t) - 0.017f(t)). \qquad (11)$$

In comparing the modeled performance for the 1970–1971 season (10) and the 1973–1974 season (11), the following points should be noted:

1) The additive constant is reduced from 66.5 to 55.5 s. Presumably this shows the change in level of performance of the swimmer at the start of the 1973–1974 season compared to the 1970–1971 season and represents an improved level of fitness before the training starts.

2) The effect of the fatigue function $f(t)$ on performance is almost the same in both seasons (multiplying constants are 0.017 and 0.015). This is not necessarily what would be expected since the swimmer is considerably more fit in the later season.

3) The effect of fitness function $p(t)$ on performance is about four times less in the later season (0.0017 compared to 0.0075). This indicates that training has less effect on the fitness or "ability to perform" of the swimmer who is already very fit.

Discussion and Conclusions

The most interesting feature of the swimmer model is the interplay between the fitness and fatigue functions in determining performance. Not only is the fatigue impulse response found to have a surprisingly long time constant (15 days), but fatigue has an unexpectedly dominant effect on performance. It is indicative of the limit to achievement from a given training environment that in the swimmer's fourth season of university training, the effect of the fitness function on performance was reduced to about one quarter of its effect in the first season. Thus as the swimmer's level of fitness nears the upper limit of his genetically determined performance capacity in any given training environment, the fatigue function becomes more and more dominant in determining performance.

The interplay between the fitness and fatigue functions is also responsible for the large rise in performance which the model predicts after training ceases (i.e., after the nineteenth week of the 1970–1971 season (Fig. 8) and after the fifteenth week of the 1973–1974 season (Fig. 10). This is a direct consequence of the relatively rapid decay of fatigue. The increase in performance when training is reduced is "well known" to coaches, and there are many qualitative discussions in the sports science literature (e.g., Zauner and Reese [21]) on how training should be tapered. Quantitative systems models should lend some precision to this discussion.

While our major interest in this paper has been to discuss the framework for a model to describe the effects of training on performance within one season of 4–6 months, it is also interesting to consider the modification of human performance over longer periods of time. For example, the

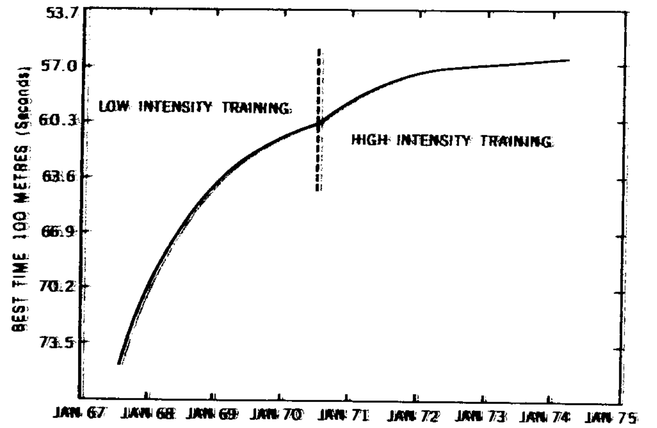

Fig. 11. Annual best performances of swimmer from 1967 to 1974.

best time in each season from 1967 to 1974 for the swimmer studied in this paper are plotted in Fig. 11. These indicate that while his performance within each season was described by the models discussed above, his long-term performance is described by a simple transfer function of the form:

$$G(s) = \frac{K}{s + (1/\tau)}$$

with the equivalent impulse response

$$g(t) = Ke^{-t/\tau}$$

and the underlying differential equation

$$\frac{dp(t)}{dt} + \frac{1}{\tau}p(t) = Kw(t). \qquad (12)$$

For this situation, the time constant is rather more than a year. It appears that his "forcing function" $w(t)$ underwent a step increase when he moved from high school in Calgary to university in Vancouver.

The application of systems theory to model the effects of physical training on performance shows that the approach is promising, but that before these methods can be applied to a wide range of activities, further investigation is required. As more detailed and better quantified data on training becomes available, it should be possible to develop more sophisticated models which have predictive ability. The models are applicable not only to the performance of international athletes but also to exercise therapy for cardiac rehabilitation and the "jogging" of the average citizen involved in supervised or self-administered fitness programs. In the knowledge that few of the readers of this paper are athletes but many are "joggers," we have included an appendix on how to quantify training and performance and on how to implement easily the prediction equations with a simple hand or desk calculator.

Appendix

Data Collection and Prediction Calculations

Since many readers have access to data from themselves or others engaged in supervised or unsupervised fitness programs, this appendix has been written to assist them in data collection and in carrying out the prediction calculations with a simple calculator. Those not currently involved in a fitness program should consult a physician before engaging in unusual activity.

A. Calibration of Training

It is difficult to describe quantitatively any human exercise which is not carried out under controlled conditions. This is particularly true for jogging over mixed terrain at variable speeds, which is the major component of many fitness programs. It has been shown that for submaximal exercise, heart rate is a fairly reliable indicator of exercise stress in a trained subject (Saltin, [9]). When exercise commences there is a heart rate transient which lasts for several minutes. Thus an exercise level can be calibrated by noting the steady-state heart rate which it produces after, let us say, ten minutes. The training impulse can then be determined by the total time for which this heart rate is maintained. Thus a heart rate of 125 beats/min maintained for 15 min produces half the training impulse which would result from maintaining it for 30 min, provided the subject is not approaching exhaustion. The most reliable technique is to exercise always at the same heart rate. However, in rough terms, a heart rate of 140 beats/min might indicate twice the stress of a heart rate of 120 beats/min. Good accuracy can be obtained by running a calibration on a bicycle ergometer and plotting a curve of power versus steady-state heart rate. The advantage of using heart rate as an indicator of exercise stress is that the work loads which cause the same stress naturally increase as the subject becomes more fit.

B. Simple Prediction Calculations

The systems models in this paper were specified in terms of transfer functions, differential equations, and convolution integrals. The solutions were simulated using a special discrete simulation package in APL on an IBM 370/155. In fact, the calculations can be performed easily and quickly on any small calculator which can perform multiplication.

The model equations (8) and (9) involve the convolution of the fitness and fatigue impulse response functions ($g_2(t)$ and $h(t)$, respectively) with the time function of training $w(t)$ which consists of a time series of impulses. Since $h(t)$ is a simple exponential (e^{-t/τ_3}) and $g_2(t)$ is the difference between two exponentials ($e^{-t/\tau_1} - e^{-t/\tau_2}$), it is only necessary to have an algorithm to convolute the time series of training impulses $w(t)$ with an exponential function. Fortunately, this reduces to a simple recursive equation.

The continuous convolution equation for the fatigue function is

$$f(t) = h(t) * w(t) = \int_0^t h(t - t')w(t')\,dt' \qquad \text{(A1)}$$

where the impulse response $h(t) = e^{-t/\tau_3}$.

If this is written in discrete form, step k corresponds to time $t = k\Delta t$, where Δt is the time step. Then

$$f(k) = \Delta t \sum_{i=1}^{k} h(k - i)w(i) \qquad \text{(A2)}$$

where

$f(k)$ the fatigue at time $k\Delta t$,
$w(k)$ the training impulse at time $k\Delta t$,
$h(k)$ the sample of the impulse response at time $k\Delta t$,

i.e., $h(k) = e^{-(k\Delta t/\tau_3)}$. Note that it follows from (A2) that

$$f(k + 1) = \Delta t \sum_{i=1}^{k+1} f(k + 1 - i)w(i). \qquad \text{(A3)}$$

Equation (A3) reduces to

$$f(k + 1) = \Delta t \cdot w(k + 1) + e^{-\Delta t/\tau_3} f(k) \qquad \text{(A4)}$$

i.e., (fatigue on day $k + 1$) = (training on day $k + 1$) + (fatigue on day k) × e^{-1/τ_3}, if the time step size $\Delta t = 1$ day.

Example

Assume 1) initial conditions are zero and 2) training is as follows.

Day	Training in ATU's
1	10
2	0
3	0
4	20
5	20
6	0
7	0
8	10
9	10
10	0

The fatigue function $f(t)$ is assumed to have a time constant of 15 days. Then $e^{-1/15} = 0.9355$.

Fatigue on day 1 = 10.

Fatigue on day 2 = $0 + 10 \times e^{-1/15} = 0 + 10 \times 0.9355 = 9.36$.

Fatigue on day 3 = $0 + 9.36 \times 0.9355 = 8.75$.

Fatigue on day 4 = $20 + 8.75 \times 0.9355 = 20 + 8.19 = 28.19$.

Fatigue on day 5 = $20 + 28.19 \times 0.9355 = 20 + 26.37 = 46.37$.

Fatigue on day 6 = $0 + 46.37 \times 0.9355 = 43.39$.

Fatigue on day 7 = $0 + 43.39 \times 0.9355 = 40.58$.

Fatigue on day 8 = $10 + 40.58 \times 0.9355 = 10 + 37.96 = 47.96$.

Fatigue on day 9 = $10 + 47.96 \times 0.9355 = 10 + 44.87 = 54.87$

Fatigue on day 10 = $0 + 54.87 \times 0.9355 = 51.33$.

In this way a profile of fatigue $f(t)$ is found, and a fitness profile $p(t)$ is calculated similarly from the difference of two exponential impulse responses but with time constants of 50 and 5 days, respectively. Performance can then be modeled using (8). Since the time constants of the model are long compared to the computation time, it is quite practical for any reader with access to a simple calculator to model his own performance in "real time."

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