1. Consider the hyperbolic triangle with vertices \(A(0,1), B(0,2)\) and \(C(1/2, \sqrt{7}/2)\). Note: this means that each pair of points is joined by a hyperbolic geodesic.

(a) Calculate the interior angles (between the geodesics) at the three vertices.

(b) How does the sum of these three angles compare with \(\pi\), in particular, is it greater than, less than, or equal to \(\pi\)? (Use your calculator.)

2. Give a precise description of a hyperbolic geodesic which passes through \((0,2)\) and makes an angle of \(\pi/3\) with the straight geodesic \(x = 0\). (The best way to describe a bowed geodesic is to give its Euclidean center and radius.)

3. Consider the circle in \(H^2\) with Euclidean center \((2,6)\) and Euclidean radius 2. Its equation is \((x - 2)^2 + (y - 6)^2 = 4\). What are its hyperbolic center and hyperbolic radius? Follow these steps:

(a) Label as \(A\) and \(B\) the points where the circle meets the vertical line \(x = 2\). Calculate the hyperbolic distance from \(A\) to \(B\). The hyperbolic radius of the circle is \(1/2\) this distance. Calculate it.

(b) Find a point \(C = (2,c)\) on the vertical line from \(A\) to \(B\), which divides this vertical line into two segments of equal hyperbolic length. (The hyperbolic lengths of these pieces will be the hyperbolic radius you calculated in (a).) This point \(C\) is the hyperbolic center, i.e., all points on the circle have the same hyperbolic distance to \(C\).

4. Given a circle in \(H^2\) with hyperbolic center \((0,5)\) and hyperbolic radius 1,000, what are its Euclidean center and Euclidean radius? Follow these steps: (While doing this problem, if you come upon an expression like \(e^{1000}\), just leave it that way, i.e., don’t plug it into your calculator.)

(a) Find two points \(A\) and \(B\) on the line \(x = 0\) that both have hyperbolic distance 1,000 from the point \((0,5)\). The Euclidean radius of the circle is simply \(1/2\) of the ordinary Euclidean distance between \(A\) and \(B\). Calculate this Euclidean radius.

(b) The Euclidean center \(C\) is simply the ordinary Euclidean midpoint of the segment \(AB\). Calculate this midpoint.