

Eric P. Klassen
Vita — February 2019

Personal:

Birthdate: 3 December 1958
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Education:

1982 B.A., Harvard University, Mathematics, summa cum laude
1984 M.S., Cornell University, Mathematics
1987 Ph.D., Cornell University, Mathematics, *Advisor:* Marshall M. Cohen

Research Grants:

1990–1993 National Science Foundation Postdoctoral Research Fellowship (\$75K) (PI)
Summer, 1992 COFRS Grant, Florida State University (\$8K) (PI)
1/95–12/97 National Science Foundation Research Grant (\$41K) (PI)
Summer, 1997 COFRS Grant, Florida State University (\$8K) (PI)
8/2001–8/2004 NSF FRG Grant (\$522K) (with A. Srivastava, D. Banks, and G. Erlebacher) (co-PI)
9/2003–8/2004 NSF ACT Grant (\$100K) (with W. Mio, X. Liu, and A. Srivastava) (co-PI)
7/2004–6/2009 USARO Grant (\$372K) (with Srivastava, Liu, and Mio) (co-PI)
8/2004–7/2007 NSF Grant (\$200K) (with D. Banks) (co-PI)
9/2009–9/2012 NSF Grant (\$400K) (with Srivastava and Barbu) (co-PI)
9/2014–9/2019 Simons Foundation Collaboration Grant (\$35K) (PI)

Positions Held:

1987–1990 Bantrell Postdoctoral Research Fellow, California Institute of Technology
1990–1995 Assistant Professor, Florida State University
1995–2002 Associate Professor, Florida State University
2002–present Professor, Florida State University
2004–2006 Associate Chair for Graduate Studies, FSU Department of Mathematics
2013–2015 Consultant, Tricircle Consulting Company

Award:

2013 University Teaching Award

Visiting Positions:

1990–91 Visiting Scholar (NSF postdoc), University of California at San Diego
Summer '93 Visiting Research Associate, California Institute of Technology
July '94 Visiting Professor, Frankfurt University (Germany)
1994–95 Visiting Assistant Professor, Indiana University

Book Authored:

2016 A. Srivastava and E. Klassen, *Functional and Shape Data Analysis*, Springer

Other Publications: (each followed by a brief summary)

★ All papers listed below have been published, are in press, or have been submitted to refereed journals and conference proceedings, except for one encyclopedia entry.

Published (in refereed journals and conference proceedings):

- (1) Wang, Sinha, and Klassen, *Exploiting topological and geometric properties for selective subdivision*, Proceedings of the Symposium on Computational Geometry (1985: Baltimore) [New York, NY]: Association for Computing Machinery, SIGGRAPH, 39–45.
In this paper, a collaboration with two applied mathematicians, we prove a theorem involving the topology of surfaces in 3-space, and use it to describe a computer algorithm for calculating the intersection of two surfaces in 3-space.
- (2) E. Klassen, *An open book decomposition for $RP^2 \times S^1$* , Proceedings of the AMS vol. 96 (1986), pp. 523–524.
In this paper I describe explicitly an open-book decomposition whose existence had been demonstrated by Berstein and Edmonds.
- (3) P. Kirk and E. Klassen, *Chern-Simons invariants of 3-manifolds and representation spaces of knot groups*, Math. Annalen 287(1990), 343–367.
If K is a knot in a 3-manifold M , we give a formula for the difference between the Chern-Simons invariants of two flat $SU(2)$ -connections on M in terms of an integral along the space of representations of the complement of K ; we then use this formula to make several explicit computations.
- (4) E. Klassen, *Representations of knot groups in $SU(2)$* , Transactions of the AMS (2)326(1991), 795–828.
In this paper, which is a published version of my PhD thesis, I calculate specifically the representation spaces of several classes of knot groups in $SU(2)$. I also prove theorems relating the dimension of these spaces to the existence of incompressible surfaces in the knot complements, and prove that those reducible representations which are singular correspond to roots of the Alexander polynomial.
- (5) P. Kirk and E. Klassen, *Representation spaces of Seifert-fibered homology 3-spheres*, Topology (1) 30 (1991), 77–95.
In this paper, we prove a conjecture of Fintushel and Stern that the representation space of every Seifert-fibered homology 3-sphere is a manifold which admits a Morse function with only even index critical points. Combining this theorem with work of Fintushel and Stern, we give a technique for computing the Floer homology of all Seifert-fibered homology 3-spheres.
- (6) C. Frohman and E. Klassen, *Deforming representations of knot groups in $SU(2)$* , Comment. Math. Helvetici 66(1991), 340–361.
We prove that those reducible representations of knot groups which correspond to simple roots of the Alexander polynomial can be deformed to arcs of irreducible representations in $SU(2)$ and in $SL(2, R)$.
- (7) E. Klassen, *Representations in $SU(2)$ of the fundamental groups of the Whitehead link and of doubled knots*, Forum Math. 5(1993), 93–109.
Using the $SU(2)$ -representation space of the Whitehead link complement computed in my thesis, I

give a method for computing the representation spaces of the twisted Whitehead doubles of certain knots.

- (8) P. Kirk and E. Klassen, *Chern-Simons invariants of 3-manifolds split along tori and the circle bundle over the representation space of T^2* , Comm. Math. Phys. 153 (1993) 521–557.

Using an explicitly constructed circle bundle over the representation space of the torus, we interpret the Chern-Simons invariant of a 3-manifold whose boundary consists of tori as a section of a bundle, and then prove an addition formula for Chern-Simons invariants of 3-manifolds which are split along tori.

- (9) B. Fine, P. Kirk and E. Klassen, *A local analytic splitting of the holonomy map on flat connections*, Math. Annalen, 299 (1994) 171-189.

Given a representation ρ of the fundamental group of a manifold M into $U(n)$, we define an analytic map from a neighborhood of ρ in the space of all such representations to the space of flat connections on M which is a 1-sided inverse of the holonomy map.

- (10) P. Kirk and E. Klassen, *Computing spectral flow via cup products*, J. Diff. Geom. 40 (1994), 505–562.

Given a path of flat connections on a 3-manifold M with boundary, we show that the first order behavior of those eigenvalues of the signature operator which are passing through zero can be calculated using a bilinear form constructed from the cohomology algebra of M . We use this to prove many cases of a conjecture of L. Jeffrey concerning spectral flow of torus bundles.

- (11) P. Kirk, E. Klassen, and D. Ruberman, *Splitting the spectral flow and the Alexander matrix*, Comment. Math. Helv. 69 (1994), 375–416.

Let X be a 3-manifold and let Y be a manifold obtained from X by replacing a solid torus in X by a knot complement. We show that the spectral flow between two flat connections on Y which are reducible on the knot complement differs from the spectral flow between the corresponding two flat connections on X by a twisted signature of the Alexander matrix of the knot.

- (12) P. Kirk and E. Klassen, *Analytic deformations of the spectrum of a family of Dirac operators on an odd-dimensional manifold with boundary*, Memoirs AMS (1995) Vol. 124, No. 592.

Given a Dirac operator on an odd-dimensional manifold with boundary, we study two types of spectrum: the extended L^2 spectrum and the Atiyah-Patodi-Singer spectrum, both of which depend on the choice of a Lagrangian subspace of the kernel of the tangential operator. Given an analytic path of such Dirac operators, we prove that the deformation theories of these two types of spectra are equivalent in an important sense.

- (13) M. Heusener and E. Klassen, *Deformations of dihedral representations*, Proc. AMS (1997) Vol. 125 No. 10, 3039-3047.

Given a representation of a knot group into $SO(3, R)$ whose image lies in a dihedral subgroup, we give a sufficient condition for this representation to lie on a smooth arc of representations of the knot group into $SO(3, R)$.

- (14) P. Kirk and E. Klassen, *The spectral flow of the odd signature operator and higher Massey products*, Math. Proc. Camb. Phil. Soc. (1997) **121**, 297-320.

Given a path of flat connections on a closed odd-dimensional manifold M , we give a technique for calculating the first non-vanishing derivatives of all those eigenvalues of the signature operator which pass through 0 at a given time. Our technique involves bilinear forms defined in terms of cup products and higher Massey products on the cohomology of M .

- (15) P. Kirk and E. Klassen, *Continuity and analyticity of families of self-adjoint Dirac operators on a manifold with boundary*, Illinois J. Math. (1998) Vol. 42, Issue 1, 123-138.

In this paper we prove basic results about the continuity and analyticity of a path of formally self-

adjoint operators with Atiyah-Patodi-Singer boundary conditions on a manifold with boundary.

- (16) M. Fried, E. Klassen and Y. Kopeliovich, *Realizing the alternating groups as monodromy groups of meromorphic functions on genus one Riemann surfaces*, Proc. AMS (2000), Vol. 129, No. 1, 111-119.

In this paper it is proven that for each $n \geq 4$, a generic Riemann surface of genus 1 admits a meromorphic function whose monodromy group is the alternating group A_n and all of whose critical points have multiplicity precisely 3.

- (17) P. Kirk and E. Klassen, *The first-order spectral flow of the odd signature operator on a manifold with boundary*, Topology and its Applications (2001), vol. 116 (2), 199-226.

Given a path of Dirac operators associated to a path of flat connections on a manifold with boundary, we show how to compute the first order behavior of those eigenvalues which pass through 0 at each point on the path. We use Atiyah-Patodi-Singer boundary conditions to make the operators self-adjoint.

- (18) H. Boden, C. Herald, P. Kirk and E. Klassen, *Gauge theoretic invariants of Dehn surgeries on knots*, Geometry and Topology, Vol. 5 (2001) Paper no. 6, 143-226.

We show how to compute several gauge theoretic invariants for manifolds obtained by Dehn surgery on knots. These invariants include spectral flow, Atiyah-Patodi-Singer rho invariants, and Boden and Herald's $SU(3)$ -version of the Casson invariants. We carry out these computations for surgeries on $(2,q)$ -torus knots.

- (19) A. Srivastava and E. Klassen, *Monte Carlo extrinsic estimators for manifold valued parameters*, IEEE Transactions on Signal Processing (2002), vol. 50, no. 2, 299 - 308.

We focus on the use of Monte Carlo methods in signal/image processing scenarios where the underlying parameter spaces are certain Riemannian manifolds. We investigate the issue of estimating means and variances of manifold-valued parameters, using ideas from independent and importance sampling. By involving the underlying geometry of these spaces, we specify the notion of extrinsic means, derive Monte Carlo methods to estimate them, and utilize large-sample asymptotics to approximate the estimator covariances, in appropriate vector spaces. The results are illustrated using applications in target pose estimation (orthogonal groups) and subspace estimation (Grassmann manifolds). The asymptotic covariances are utilized to construct confidence regions, to perform comparisons, and to determine the sample size for MC methods.

- (20) A. Srivastava, W. Mio, X. Liu, and E. Klassen, *Computational Approaches to a Statistical Theory of Shapes*, Proc. 17th Annual Conf. on Neural Information Processing Systems, Vancouver, Canada, 2003.

Need Abstract Here.

- (21) A. Srivastava, W. Mio, E. Klassen, and X. Liu, *Geometric Analysis of Constrained Curves for Image Understanding*, Proc. 2nd IEEE Workshop on Variational, Geometric and Level-Set Methods in Computer Vision, Nice, France, 2003.

We offer an overview of a novel approach to the computational differential geometry of spaces of curves with applications to shape analysis and discovery of objects in images. Applications include the statistical analysis of planar shapes, completion of curves with elasticae, and Bayesian discovery of contours of objects in noisy images.

- (22) A. Srivastava, W. Mio, E. Klassen, and S. Joshi, *Geometric Analysis of Continuous, Planar Shapes*, Proc. 4th International Workshop on Energy Minimization Methods in Computer Vision and Pattern Recognition, Lisbon, Portugal, 2003.

We propose two differential geometric representations of planar shapes using: (i) direction functions and (ii) curvature functions of their boundaries. Under either representation, planar shapes are treated as elements of infinite-dimensional shape spaces. Differences between shapes are quantified using lengths of geodesics connecting them in shape spaces. We specify the geometry of the

two shape spaces and utilize numerical methods to compute geodesics. Applications include: (i) interpolation and extrapolation of shapes, (ii) clustering of objects according to their shapes, and (iii) computation of intrinsic mean shapes.

- (23) A. Srivastava and E. Klassen, *Bayesian and Geometric Subspace Tracking*, J. Advances in Applied Probability 36(1) 2004, p. 43-56.

We address the problem of tracking principal subspaces using ideas from nonlinear filtering. The subspaces are represented by their complex projection matrices and time-varying subspaces correspond to trajectories on the Grassmannian manifold. Under a Bayesian approach, we impose a smooth prior on the velocities associated with the subspace motion. This prior combined with any standard likelihood function forms a posterior density on the Grassmannian, for filtering and estimation. Using a sequential Monte Carlo method, a recursive nonlinear tracking algorithm is derived and some implementation results are presented.

- (24) E. Klassen, A. Srivastava, W. Mio, and S. Joshi, *Analysis of Planar Shapes Using Geodesic Paths on Shape Manifolds*, IEEE Trans. Pattern Analysis and Machine Intelligence, 26(3) 2004, p. 372-384.

A manifold structure is proposed for the space of all closed planar curves of rotation index 1 modulo rigid motion and rescaling. A numerical algorithm is given for producing geodesics between any two points on this manifold. The length of such a geodesic gives a quantitative measure of “how different” two planar shapes are. As an application, means and variances are calculated for collections of shapes, and random distributions of shapes are produced.

- (25) E. Klassen and Y. Kopeliovich, *Hurwitz spaces and braid group representations*, Rocky Mountain Journal of Mathematics, 34(3) 2004, p. 1005-1031.

Associated to each branched cover of the Riemann sphere there is a Hurwitz space, and a corresponding representation of a finite index subgroup of the spherical braid group. In this paper we show how to compute this representation explicitly, and use this computation to show that the generic Riemann surface of genus one admits a meromorphic function with certain prescribed combinatorial branch structure.

- (26) W. Mio, A. Srivastava, and E. Klassen, *Interpolations with Elasticae in Euclidean Spaces*, Quarterly of Applied Mathematics, 67(2) 2004, p. 359-378.

Algorithms are given for producing “least energy” curves in n -dimensional Euclidean spaces satisfying first order boundary conditions. The types of energy minimized include elastic energy, scale-invariant elastic energy, and a linear combination of elastic energy and length. Applications to completing partially occluded images are discussed.

- (27) E. Klassen and A. Srivastava, *Geodesics Between 3D Closed Curves Using Path Straightening*, Proceedings of ECCV, Lecture Notes in Computer Science, 2006, p. 95-106.

In order to analyze shapes of continuous curves in R^3 , we parameterize them by arc-length and represent them as curves on a unit two-sphere. We identify the subset denoting the closed curves, and study its differential geometry. To compute geodesics between any two such curves, we connect them with an arbitrary path, and then iteratively straighten this path using the gradient of an energy associated with this path. The limiting path of this path-straightening approach is a geodesic. Next, we consider the shape space of these curves by removing shape-preserving transformations such as rotation and re-parametrization. To construct a geodesic in this shape space, we construct the shortest geodesic between the all possible transformations of the two end shapes; this is accomplished using an iterative procedure. We provide step-by-step descriptions of all the procedures, and demonstrate them with simple examples.

- (28) S. Joshi, E. Klassen, A. Srivastava, and I. Jermyn, *An Efficient Representation for Computing Geodesics Between n -Dimensional Elastic Shapes*, IEEE Conference on Computer Vision and Pattern Recognition (CVPR), June 2007.

We propose an efficient representation for studying shapes of closed curves in R^n . This paper combines the strengths of two important ideas - elastic shape metric and path-straightening methods - and results in a very fast algorithm for finding geodesics in shape spaces. The elastic metric allows for optimal matching of features between the two curves, while path-straightening ensures that the algorithm results in geodesic paths. For the novel representation proposed here, the elastic metric becomes the simple L^2 metric, in contrast to the past usage where more complex forms were used. We present the step-by-step algorithms for computing geodesics and demonstrate them with 2-D as well as 3-D examples.

- (29) S. Joshi, A. Srivastava, E. Klassen, and I. Jermyn, *Removing Shape-Preserving Transformations in Square-Root Elastic (SRE) Framework for Shape Analysis of Curves*, Proceedings of the Workshop on Energy Minimization Methods in CVPR, p. 387-398, August 2007.

We illustrate and extend the “Square Root Elastic” (SRE) method of comparing closed curves in 2D or 3D space. This framework allows for elastic comparison (allowing both stretching and bending), and also utilizes a path straightening algorithm to compute geodesics efficiently in shape space. The method works in spaces of arbitrary dimension.

- (30) C. Samir, A. Srivastava, M. Daoudi, and E. Klassen, *An Intrinsic Framework for Analysis of Facial Surfaces*, International Journal of Computer Vision 82(1), p. 80-95, April 2009.

We extend earlier work on comparing closed curves in 3D space to provide a method of comparing Facial Surfaces.

- (31) C. Samir, P.-A. Absil, A. Srivastava and E. Klassen, *Fitting Curves on Riemannian Manifolds Using Energy Minimization*, Proceedings of the IAPR Conference on Machine Vision Applications, p. 422-425, Yokohama, Japan 2009.

We devise an algorithm for finding a path on a Riemannian manifold that is a piecewise geodesic, and that also comes close to a given sequence of points at given values of the parameter. This path is defined by minimizing an energy functional that is a sum of a regularity term, and a term which involves the distances to the given points at the given parameter values. We use a gradient search method for finding the path.

- (32) S. Kurtek, E. Klassen, Z. Ding, and A. Srivastava, *A Novel Riemannian Framework for Shape Analysis of 3D Objects*, IEEE Conference on computer Vision and Pattern Recognition (CVPR), San Francisco, CA, June 2010.

- (33) S. Kurtek, E. Klassen, A. Srivastava, Z. Ding, S. W. Jacobson, J. L. Jacobson, and M. J. Avison, *A Novel Parameterization-Invariant Riemannian Framework for Comparing Shapes of 3D Anatomical Structures*, 10 pages, accepted as a poster presentation in International Society for Magnetic Resonance in Medicine, Stockholm, Sweden, May 2010.

- (34) A. Srivastava, E. Klassen, S. Joshi, and I. Jermyn, *Shape Analysis of Elastic Curves in Euclidean Spaces*, IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 33, issue 7, pages 1415-1428, July 2011.

We develop a Riemannian structure on the space of curves in R^n , where elastic matching is allowed between curves, using the square root velocity (SRV) representation. This structure allows precise comparison, classification, and statistical analysis of these curves.

- (35) C. Samir, P.-A. Absil, A. Srivastava, and E. Klassen, *A Gradient-Descent Method for Curve Fitting on Riemannian Manifolds*, Foundations of Computational Mathematics, vol. 12, issue 1, p. 49-73, 2012.

Given data points p_0, \dots, p_N on a manifold \mathcal{M} and time instants $0 = t_0 < t_1 < \dots < t_N = 1$, we consider the problem of finding a curve γ on \mathcal{M} that best approximates the data points at the given instants while being as “regular” as possible. Specifically, γ is expressed as the curve that minimizes the weighted sum of a sum-of-squares term penalizing the lack of fitting to the data points and

a regularity term defined, in the first case as the mean squared velocity of the curve, and in the second case as the mean squared acceleration of the curve. In both cases, the optimization task is carried out by means of a steepest-descent algorithm on a set of curves on \mathcal{M} . The steepest-descent direction, defined in the sense of the first-order and second-order Palais metric, respectively, is shown to admit simple formulas.

- (36) Z. Zhang, E. Klassen, R. Chellappa, P. Puraga, and A. Srivastava, *Blurring-Invariant Riemannian Metrics for Comparing Images and Signals*, International Conference on Computer Vision (ICCV), Barcelona, Spain, November 2011.

We propose a novel Riemannian framework for comparing signals and images in a manner that is invariant to their levels of blur. This framework uses a log-Fourier representation of signals/images in which the set of all possible blurrings of a signal, i.e. its orbit under semigroup action of a Gaussian blur function, is a straight line. Using a Riemannian metric under which the group action is by isometries, the orbits are compared via distances between orbits. We demonstrate this framework using a number of experimental results involving 1D signals and 2D images.

- (37) S. Kurtek, Z. Ding, E. Klassen, and A. Srivastava, *Parameterization-Invariant Shape Statistics and Probabilistic Classification of Anatomical Surfaces*, Information Processing in Medical Imaging (IPMI), Kloster Irsee, Germany, July 2011.

We consider the task of computing shape statistics and classification of 3D anatomical structures (as continuous, parameterized surfaces) under a Riemannian framework. This task requires a Riemannian metric that allows: (1) reparameterizations of surfaces by isometries, and (2) efficient computations of geodesic paths between surfaces. These tools allow for computing Karcher means and covariances (using tangent PCA) for shape classes, and a probabilistic classification of surfaces into disease and control classes. In addition to toy objects, we use the Detroit Fetal Alcohol and Drug Exposure Cohort data to study brain structures and present classification results for the Attention Deficit Hyperactivity Disorder cases and controls in this study. We find that using the mean and covariance structure of the given data, we are able to attain a 88% classification rate, which is an improvement over a previously reported result of 82% on the same data.

- (38) S. Kurtek, E. Klassen, Z. Ding, S. Jacobson, J. Jacobson, M. Avison, A. Srivastava, *Parameterization-Invariant Shape Comparisons of Anatomical Surfaces*, IEEE Transactions on Medical Imaging vol. 30, no. 3, pages: 849-858, 2011.

We consider 3D brain structures as continuous parameterized surfaces and present a metric for their comparisons that is invariant to the way they are parameterized. We demonstrate this method in shape analysis of multiple brain structures, for 34 subjects in the Detroit Fetal Alcohol and Drug Exposure Cohort study, which results in a 91% classification rate for ADHD (Attention Deficit Hyperactivity Disorder) cases and controls. This method outperforms some existing techniques such as SPHARM-PDM (spherical harmonic point distribution model) or ICP (iterative closest point).

- (39) S. Kurtek, E. Klassen, J. C. Gore, Z. Ding, and A. Srivastava, *Classification of Mathematics Deficiency Using Shape and Scale Analysis of 3D Brain Structures*, SPIE Conference on Medical Imaging, Orlando, FL, February, 2011.

- (40) S. Kurtek, E. Klassen, J. Gore, Z. Ding, and A. Srivastava, *Elastic Geodesic Paths in Shape Space of Parameterized Surfaces*, IEEE Transactions on Pattern Analysis and Machine Intelligence, Sept, 2012, 34(9), p. 1717-1730.

This paper presents a novel Riemannian framework for shape analysis of parameterized surfaces. In particular, it provides efficient algorithms for computing geodesic paths which, in turn, are important for comparing, matching, and deforming surfaces. The novelty of this framework is that geodesics are invariant to the parameterizations and other shape-preserving transformations of surfaces. We illustrate these ideas using examples from shape analysis of anatomical structures

and other surfaces.

- (41) D. Bryner, E. Klassen and A. Srivastava, *Affine-Invariant, Elastic Shape Analysis of Planar Contours*, IEEE Computer Vision and Pattern Recognition (CVPR) conference, 2012, Providence, RI.

We give an algorithm for comparing closed planar curves up to affine equivalence, using an elastic metric.

- (42) I Jermyn, S. Kurtek, E. Klassen, A. Srivastava, *Elastic Matching of Parametrized Surfaces Using Square-Root Normal Fields*, European Conference on Computer Vision, Florence, Italy, Nov. 2012.

- (43) J. Su, L. Dryden, E. Klassen, H. Le, and A. Srivastava, *Fitting smoothing splines to time-indexed, noisy points on nonlinear manifolds*, Journal of Image and Vision Computing, 30(6-7): 428-442, 2012.

We address the problem of estimating full curves/paths on certain nonlinear manifolds using only a set of time-indexed points, for use in interpolation, smoothing, and prediction of dynamic systems. These curves are analogous to smoothing splines in Euclidean spaces as they are optimal under a similar objective function, which is a weighted sum of a fitting-related (data term) and a regularity-related (smoothing term) cost functions. The search for smoothing splines on manifolds is based on a Palais metric-based steepest-descent algorithm developed in Samir et al. Using three representative manifolds: the rotation group for pose tracking, the space of symmetric positive-definite matrices for DTI image analysis, and Kendall's shape space for video-based activity recognition, we demonstrate the effectiveness of the proposed algorithm for optimal curve fitting. This paper derives certain geometrical elements, namely the exponential map and its inverse, parallel transport of tangents, and the curvature tensor, on these manifolds, that are needed in the gradient-based search for smoothing splines. These ideas are illustrated using experimental results involving both simulated and real data, and comparing the results to some current algorithms such as piecewise geodesic curves and splines on tangent spaces, including the method by Kume et al.

- (44) S. Kurtek, A. Srivastava, E. Klassen, and Z. Ding, *Statistical Modeling of Curves Using Shapes and Related Features*, Journal of the American Statistical Association vol. 107, no. 499, pp. 1152-1165, 2012.

We develop statistical models of 3D parameterized curves in terms of combinations of features such as shape, placement, scale, and rotation. For each combination of interest, we identify a representation manifold, endow it with a Riemannian metric, and outline tools for computing sample statistics on these manifolds. The nuisance variables, including reparametrization, are removed by forming quotient spaces under appropriate group actions. In case of shape analysis, the resulting spaces are quotient spaces of Hilbert spheres, and we derive certain wrapped truncated normal densities for capturing variability in observed curves. We demonstrate these models using both artificial data and real data involving fiber tracts from multiple subjects and protein backbones from the SHREC 2010 database.

- (45) S. Kurtek, E. Klassen, A. Srivastava, and H. Laga, *Landmark-Guided Elastic Shape Analysis of Spherically-Parameterized Surfaces*, Computer Graphics Forum (Special Issue of Eurographics 2013), vol. 32, no. 2, 2013.

We enhance the method described in paper (40) above (as well as several others) in order to compare surfaces in a way that is compatible with a designated correspondence between landmarks on the two surfaces.

- (46) Z. Zhang, E. Klassen, A. Srivastava, *Gaussian Blurring-Invariant Comparisons of Signals and Images*, IEEE Transactions on Image Processing, vol. 22, no. 8, August 2013.

We present a Riemannian framework for analyzing signals and images in a manner that is invariant to their level of blurriness, under Gaussian blurring, using a well-known relation between Gaussian blurring and the heat equation.

- (47) A. Srivastava, S. Kurtek, E. Klassen, *Statistical Shape Analysis*, Encyclopedia of Computer Vision, Springer, 2012.
- (48) J. Su, S. Kurtek, E. Klassen, A. Srivastava, *Statistical Analysis of Trajectories on Riemannian Manifolds: Bird Migration, Hurricane Tracking, and Video Surveillance*, Annals of Applied Statistics, Vol. 8, No. 1, p. 530-552, 2014.
- (49) D. Bryner, E. Klassen, H. Le, A. Srivastava, *2D Affine and Projective Shape Analysis*, IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 36(5), p. 998-1011, 2014.
- (50) Q. Xie, S. Kurtek, E. Klassen, G. Christensen, A. Srivastava, *Metric-Based Pairwise and Multiple Image Registration*, European Conference on Computer Vision, Zurich, Switzerland, Proceedings, Part II, p. 236-250, 2014.
- (51) M. Rosenthal, W. Wu, E. Klassen, A. Srivastava, *Spherical Regression Models Using Projective Linear Transformations*, J. of the American Statistical Association, Vol. 109, Issue 508, p. 1615-1624, 2014.
- (52) A. Duncan, E. Klassen, X. Descombes and A. Srivastava, *Geometric Analysis of Axonal Tree Structure*, British Machine Vision Conference Proceedings, Swansea, UK, 11 pages, September 2015.
- (53) S. Lahiri, D. Robinson, and E. Klassen, *Precise Matching of PL Curves in R^n in the Square Root Velocity Framework*, Geometry, Imaging, and Computing, Vol. 2, No. 3, p. 133-186, 2015.
- (54) Z. Su, E. Klassen, and M. Bauer, *The Square Root Velocity Framework for Curves in a Homogeneous Space*, 8 pages, Diff-CVML Workshop, CVPR, July, 2017.
- (55) D. Robinson, A. Duncan, A. Srivastava, and E. Klassen, *Exact Function Alignment Under Elastic Riemannian Metric (Book Chapter)*, Graphs in Biomedical Image Analysis, Computational Anatomy and Imaging Genetics, 137-151, 2017.
- (56) Z. Zhang, E. Klassen, and A. Srivastava, *Phase-Amplitude Separation and Modeling of Spherical Trajectories*, Journal of Computational and Graphical Statistics 27 (1), 85-97, 2018.
- (57) A. Duncan, E. Klassen, and A. Srivastava, *Statistical shape analysis of simplified neuronal trees*, The Annals of Applied Statistics 12 (3), 1385-1421, 2018.
- (58) Z. Zhang, J. Su, E. Klassen, H. Le, and A. Srivastava, *Rate-Invariant Analysis of Covariance Trajectories*, Journal of Mathematical Imaging and Vision, 1-18, 2018.
- (59) Z. Su, E. Klassen, and M. Bauer, *Comparing Curves in Homogeneous Spaces*, Differential Geometry and its Applications 60, 9-32, 2018.
- (60) Z. Zhang, E. Klassen, and A. Srivastava, *Robust Comparison of Kernel Densities on Spherical Domains*, Sankhya A, 1-28, 2018.

In press (in refereed journals and conference proceedings):

Submitted (to refereed journals and conference proceedings):

- (61) M. Bauer, E. Klassen, S.C. Preston, Z. Su, *A diffeomorphism-invariant metric on the space of vector-valued one-forms*, submitted to Journal of Differential Geometry, 31 pages, 2019.
- (62) M. Rosenthal, W. Wu, E. Klassen, A. Srivastava, *Nonparametric Spherical Regression Using Diffeomor-*

phic Mappings, 18 pages, submitted to Computational Statistics and Data Analysis, February, 2019.

Citations: (other than by myself and coauthors)

1990: 1
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Invited Lectures:

Rice University, Feb. 1987
University of Pennsylvania, Feb. 1987
Columbia University, Feb. 1987
Michigan State Topology Conference, May 1987
Georgia Topology Conference, Aug. 1987
Berkeley/Stanford Topology Seminar, Oct. 1987
UCLA, Oct. 1987
AMS special session (org. by Bonahon), Claremont, CA, Nov. 1988
Georgia Topology Conference, Aug. 1989
Harvard University, Nov. 1989
Tufts University, Nov. 1989
Brandeis University, Nov. 1989
UCLA Southern California Topology Seminar, Dec. 1989
Florida State University, February 1990
Kansas State University, February 1990
University of Iowa, March 1990
University of Wisconsin, March 1990

University of Quebec at Montreal, March 1990
University of Utah, March 1990
University of Hawaii Topology conference, Aug. 1990
Georgia Topology Conference, Aug. 1991
Yale University, April 1992
University of Texas, May 1992
University of Tennessee low-dimensional topology conference, May 1992
Indiana University, June 1992
Florida State University fall topology conference, Oct. 1992
AMS special session (org. by Mrowka), Los Angeles, Nov. 1992
AMS special session (org. by Rong), Washington DC, Apr. 1993
Caltech gauge theory seminar, June 1993
Colloquium, Siegen University, Germany, July 1994
Colloquium, Frankfurt University, Germany, July 1994
Topology Seminar, Frankfurt University, Germany, July 1994
Indiana University/Purdue University at Indianapolis topology seminar, Sept. 1994
Indiana University topology seminar, Sept. 1994
Colloquium, Florida State University, Sept. 1995
AMS special session (org. by P. Gilmer), Baton Rouge, April 1996
AMS special session (org. by P. Bowers), Atlanta, October 1997
AMS special session (org. by D. Auckly), Manhattan KS, March 1998
AMS special session (org. by H. Volklein), Gainesville FL, January 1999
Colloquium, University of Helsinki, Finland, May 1999
AMS special session (org. by M. Seppala), New Orleans, January 2001
Colloquium, Florida State University, Oct. 2001
UF-FSU Topology and Geometry Conference, Tallahassee, Feb. 2003
UF-FSU Topology and Geometry Conference, Gainesville, Dec. 2006
“Shape Day” Workshop, FSU, April, 2007
Workshop, Lille University, Lille, France, July, 2008
Seminar, INRIA, Antibes, France, July, 2008.
Seminar, SAMSI, Research Triangle Park, NC, October, 2010.
3rd Conference of Tsinghua Sanya International Mathematics Forum, Sanya, China, Jan. 2013
FSU Statistics Department, Weekly Shape Seminar, September 16, 2014.
FSU Math Fun Day Lecture, October 11, 2014.
University of Nottingham, UK, February 12, 2015.
Schroedinger Institute, Vienna, Austria, February 18, 2015.
Schroedinger Institute, Vienna, Austria, February 26, 2015.
Norwegian University of Science and Technology, Trondheim, Norway, September 29, 2015.
SIAM Special Session, Pittsburgh, PA, July 14, 2017
FSU Workshop on Applications-Driven Geometric Functional Data Analysis Tutorial October 8, 2017
FSU Workshop on Applications-Driven Geometric Functional Data Analysis Research Talk October 9, 2017
AMS Special Session, Columbus, Ohio, March 17, 2018

CBMS Workshop, Columbus OH, “Elastic Functional and Shape Data Analysis” , July 16-20, 2018. (two talks)

Harvard University Center of Math. Sci. and Applications Workshop on Morphometrics, Geometry, and Statistics, Oct. 22-24, 2018. (one talk)

McMaster University Dept. of Math. Colloquium, November 16, 2018.

Banff (BIRS) Workshop on Shape Analysis, Stochastic Mechanics, and Optimal Transport, Dec. 9-14, 2018. (one talk)

University of Ottawa Colloquium, March 22, 2019.

Meeting Organized:

AMS special session, Charlotte, NC, October 1999

Refereeing:

Transactions of the AMS (1988,2000)

Proceedings of the Georgia International Topology Conference (1993,2002)

NSF Proposals (1993-2004)

Proceedings of the AMS (1995)

Communications in Algebra (1996)

Comment. Math. Helvetici (1998)

Proc. of the London Math. Soc. (1998,2000)

Modern Physics Letters A (2000)

Topology and Its Applications (2001)

Annals of Applied Probability (2006)

Austrian Science Fund (2012)

SIAM Journal on Mathematical Analysis (2015)

Journal of Mathematical Imaging and Vision (2017)

Teaching (courses taught):

Fall ‘91 MAD 3104 (Discrete Math I)
 MAC 3311 (Calculus I)

Fall ‘92 MAD 3105 (Discrete Math II)
 MTG 4302/5316 (Elementary Topology)

Spring ‘93 DIS in number theory

Fall ‘93 MAC 3311 (Calculus I)
 MTG 5326 (Topology I)

Spring ‘94 MAC 3312 (Calculus II, honors)
 MTG 5327 (Topology II)

Fall ‘94 On leave

Spring '95	On leave
Fall '95	MAC 3313 (Calculus III)
Spring '96	MAC 3311 (Calculus I) MTG 4212 (College Geometry)
Fall '96	MTG 4212 (College Geometry) MTG 6396 (Topics in Topology)
Spring '97	MAD 3104 (Discrete Math I) MTG 6396 (Topics in Topology)
Summer '97	MAD 3105 (Discrete Math II)
Fall '97	MTG 4212 (College Geometry) MTG 5376 (Topics in Topology) MTG 6939 (Topology Seminar)
Spring '98	MAC 2311 (Calculus I) MTG 6939 (Topology Seminar)
Fall '98	MAC 2311 (Calculus I) MTG 5316 (Elementary Topology) MTG 6939 (Topology Seminar) MTG 3930 (Putnam Preparation)
Spring '99	MTG 4212 (College Geometry) MTG 6939 (Topology Seminar)
Fall '99	MTG 4212 (College Geometry) MAC 2312 (Calculus II, honors)
Spring '00	MAC 2313 (Calculus III)
Summer '00	MAC 2312 (Calculus II)
Fall '00	On Sabbatical
Spring '01	MTG 2312 (Calculus II, honors) MGF 3301 (Introduction to Advanced Mathematics)
Summer '01	MAC 2313 (Calculus III)
Fall '01	MTG 4212 (College Geometry) MAT 5932 (Differential Geometry)
Spring '02	MAC 2312 (Calculus II, two sections)
Fall '02	MTG 4212 (College Geometry) MAC 2313 (Calculus III)

Spring '03	MAC 2313 (Calculus III)
Fall '03	MTG 4212 (College Geometry) MAC 2312 (Calculus II)
Spring '04	MAC 2313 (Calculus III, two sections)
Fall '04	MTG 4212 (College Geometry)
Fall '05	MTG 4212 (College Geometry)
Fall '06	MTG 4212 (College Geometry)
Spring '07	MAC 2311 (Calculus I) MTG 5932 (Differential Topology) MTG 6939 (Topology Seminar)
Summer '07	MTG 5932 (Lie Groups)
Fall '07	MTG 4212 (College Geometry) MTG 5346 (Algebraic Topology I)
Spring '08	MTG 5347 (Algebraic Topology II) MAC 2313 (Calculus III)
Fall '08	MTG4212 (College Geometry) MTG5932 (Differential Topology) MAT6908 (DIS - Algebraic Topology)
Spring '09	MAC2313 (Calculus III) MAT6908 (DIS - Characteristic Classes and Index Theory)
Fall '09	MTG4212 (College Geometry) MTG5932 (Group Representation Theory)
Spring '10	MAP4153 (Vector Calculus)
Fall '10	MTG4212 (College Geometry) MAC2313 (Calculus III)
Spring '11	MTG5932 (Riemannian Geometry)
Summer '11	MAA4402 (Complex Variables)
Fall '11	MTG4212 (College Geometry)
Spring '12	MAA4402 (Complex Variables) MTG4302 (Elementary Topology)
Fall '12	MTG4212 (College Geometry) MAA4226 (Advanced Calculus I)
Spring '13	MAA4227 (Advanced Calculus II)

	MAC2312 (Honors Calculus II)
Summer '13	MAA4402 (Complex Variables)
Fall '13	MTG4212 (College Geometry) MTG5326 (Topology I)
Spring '14	MTG5327 (Topology II)
Summer '14	MAA4227 (Advanced Calculus II)
Fall '14	MTG4212 (College Geometry) MAC2313 (Calculus III)
Spring '15	On Sabbatical
Summer '15	MAA4402 (Complex Variables) MAT5907 (DIS on Differential Geometry of Curves and Surfaces)
Fall '15	MTG4212 (College Geometry) MAA4226 (Advanced Calculus I)
Spring '16	MAA4227 (Advanced Calculus II) MTG5932 (Riemannian Geometry)
Fall '16	MTG4212 (College Geometry) MAA4224 (Introduction to Analysis I)
Spring '17	MTG4302 (Elementary Topology I)
Summer '17	MAA4402 (Complex Variables)
Fall '17	MTG4212 (College Geometry) MTG5346 (Algebraic Topology)
Spring '18	MTG4302 (Elementary Topology I)
Fall '18	MTG4212 (College Geometry) MAA4402 (Complex Variables)
Spring '19	MAC2313 (Calculus III)