Errata for Implementing Spectral Methods: Algorithms for Scientists and Engineers

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1 Chapter 1

1. Page 7. Eq. (1.28) should be

$$\begin{aligned}\|f(x) - P_N f(x)\|^2 &= \int_0^{2\pi} \left(\sum_{|k|=N/2+1}^{\infty} \hat{f}_k e^{ikx} \right) \left(\sum_{|l|=N/2+1}^{\infty} \hat{f}_l^* e^{-ilx} \right) dx \\ &= \sum_{|k|=N/2+1}^{\infty} \sum_{|l|=N/2+1}^{\infty} \hat{f}_k \hat{f}_l^* \int_0^{2\pi} e^{ikx} e^{-ilx} dx \\ &= \sum_{|k|=N/2+1}^{\infty} \sum_{|l|=N/2+1}^{\infty} \hat{f}_k \hat{f}_l^* \left(e^{ikx}, e^{ilx} \right) \\ &= \sum_{|k|=N/2+1}^{\infty} \left| \hat{f}_k \right|^2 \left\| e^{ikx} \right\|^2.\end{aligned}$$

   (Thanks to Yaning Liu)

2. Page 11. Eq. (1.37) on page 11 should be

$$\begin{aligned}\|f(x) - S_N\|^2 &= \left\| \sum_{k=-N/2}^{N/2} \left(\hat{f}_k - \hat{s}_k \right) e^{ikx} + \sum_{|k|=N/2+1}^{\infty} \hat{f}_k e^{ikx} \right\|^2 \\ &= 2\pi \left\{ \sum_{k=-N/2}^{N/2} \left| \hat{f}_k - \hat{s}_k \right|^2 + \sum_{|k|=N/2+1}^{\infty} \left| \hat{f}_k \right|^2 \right\}.\end{aligned}$$

   (Thanks to Ahmed Derar Islim)

3. Page 16. Eq. (1.68) The a_n and b_n should be $\tilde{a}_n$ and $\tilde{b}_n$ (Thanks to Yaning Liu)

4. Page 38, Problem 1.2. The coefficients given are for $2\pi\hat{f}_k$. (Thanks to Allan Engsig-Karup.)

2 Chapter 2

1. Page 41. Eq. (2.3) the last entry on the left should be F_{N-1}. Also, the $s = -1$ three lines above should be $s = 1$.(Thanks to Celes Woodruff)

2. Page 42. Alg. 8. In the final group of loops, the line for f_{d+i} should be

$$f_{d+i} \leftarrow W * (f_{c+i} - f_{d+i})$$

   that is, replace w with W. (Thanks to Dong Sun)

3. Page 45. In Equation (2.12), the index should be k=0,1,...M-1 instead of j. (Thanks to Yaning Liu)

4. Page 46. In equation (2.14), the exponential term should be $e^{-2\pi ik/N}$, i.e. there is no j. (Thanks to Yaning Liu)

5. Page 49. In equation (2.27), replace j with n. (Thanks to Celes Woodruff)

6. In Alg. 16, replace $X_k \leftarrow (a_k + ib_k)/2$ by $X_k \leftarrow (a_k - ib_k)/2$. (Thanks to Juan A. Calzada)

7. Page 54. In Alg. 17, replace

$$F_{-N/2} \leftarrow 0$$

   with

$$F_{N/2} \leftarrow 0$$

3 Chapter 3

1. Page 60. Algorithm 20: $L_{k-1}(x)$ should be L_{k-1}. (Thanks to James Custer)

2. Page 63. Algorithm 22:

$$L_N \leftarrow \frac{2k-1}{k} x L_{N-1}(x) - \frac{k-1}{k} L_{N-2}$$

   should read

$$L_N \leftarrow \frac{2k-1}{k} x L_{N-1} - \frac{k-1}{k} L_{N-2}$$

   (Thanks to Travis Johnson)

3. Page 63. Algorithm 22: The description says "Degree k" but should be "Degree N" and near the very bottom $L'_N(x)$ should be L'_N. (Thanks to James Custer)

4. Page 65. Algorithm 24 has a one-off error. A rewrite is shown below in Alg. 1. (Thanks to Matt Jemison)

5. Page 69. The last line should read: since periodicity of the coefficients means that $\bar{f}_0 = \bar{f}_{2N}$. (Thanks to Branden Neese.)

6. Page 71. In the line under equation (3.26) "When we substitute for f_j", the f_j should be $\bar{f}_j$. (Thanks to Yaning Liu.)

7. Page 75-76. For the sake of clarity, re-write (3.38) and (3.39) as

$$T_{kj} = \frac{\frac{w_j}{\xi_k - x_j}}{\sum_{n=0}^{N} \frac{w_n}{\xi_k - x_n}}.$$

Algorithm 1: *qAndLEvaluation:* Combined Algorithm to Compute $L_N(x)$, $q(x) = L_{N+1} - L_{N-1}$, and $q'(x)$

Procedure qAndLEvaluation
Input: N, x
Comment: Invoked only for $N \geq 2$
$L_{N-2} \leftarrow 1$
$L_{N-1} \leftarrow x$
$L'_{N-2} \leftarrow 0$
$L'_{N-1} \leftarrow 1$
for $k = 2$ **to** N **do**
  $L_N \leftarrow \frac{2k-1}{k} x L_{N-1} - \frac{k-1}{k} L_{N-2}$
  $L'_N \leftarrow L'_{N-2} + (2k-1)\, L_{N-1}$
  $L_{N-2} \leftarrow L_{N-1}$
  $L_{N-1} \leftarrow L_N$
  $L'_{N-2} \leftarrow L'_{N-1}$
  $L'_{N-1} \leftarrow L'_N$
end
$k \leftarrow N + 1$
$L_{N+1} \leftarrow \frac{2k-1}{k} x L_N - \frac{k-1}{k} L_{N-2}$
$L'_{N+1} \leftarrow L'_{N-2} + (2k-1)\, L_{N-1}$
$q \leftarrow L_{N+1} - L_{N-2}$
$q' \leftarrow L'_{N+1} - L'_{N-2}$
return q, q', L_N
End Procedure qAndLEvaluation

and

$$\ell_j(x) = \frac{w_j}{(x - x_j) \sum_{n=0}^{N} \frac{w_n}{x - x_n}}.$$

(Thanks to James Custer.)

8. Page 76. Algorithm 32: $T_{k,j} \leftarrow T_{kj}/s$ should be $T_{k,j} \leftarrow T_{k,j}/s$. (Thanks to James Custer.)

9. Page 79. Algorithm 35: EndProcedure InterpolateToNewPoints should be EndProcedure 2DCoarseToFineInterpolation (Thanks to James Custer)

10. Page 80. Algorithm 36: $p \leftarrow$ *LagrangeInterpolation*(... should not have N in the parameter list. Algorithm 31 only takes 4 parameters. (Thanks to James Custer)

11. Page 83. Algorithm 38. We need to use the previous derivative's matrix elements but its modifying the diagonal element in the loop. Before setting the diagonal element to zero, add a temporary variable, $D^{(m-1)}_{i,i} = D^{(m)}_{i,i}$, and replace the line

$$D^{(m)}_{i,j} \leftarrow \frac{k}{x_i - x_j} \left(\frac{w_j}{w_i} D^{(m)}_{i,i} - D^{(m)}_{i,j} \right)$$

with

$$D^{(m)}_{i,j} \leftarrow \frac{k}{x_i - x_j} \left(\frac{w_j}{w_i} D^{(m-1)}_{i,i} - D^{(m)}_{i,j} \right)$$

(Thanks to James Custer)

4 Chapter 4

1. Page 92. Eq. 4.6 should be φ_x in the boundary term. Should be

$$\int_0^L \varphi_t \phi dx = \int_0^L \left(\nu\varphi_x\right)_x \phi dx = \nu\varphi_x\phi|_0^L - \int_0^L \nu\varphi_x\phi_x dx.$$

2. Page 94. After eq. (4.14) $x_n = 2\pi n/N$. The n is missing on the right in the text.

3. Page 94. In equation (4.17) all indices should be i. (Thanks to Yaning Liu.)

4. Page 100. The line above (4.30) says that "...If we set $k = 0$ then (1.71) says that". Eq. (1.71) would be better replaced by (1.72). (Thanks to Yaning Liu.)

5. Page 102. The boundary term in eqs. (4.33) and (4.34) should have ϕ^*. (Thanks to David Mandel.)

6. Page 102. The boundary term should have e^{-ikx}. (Thanks to David Mandel.)

7. Page 103. Eq. (4.39) is missing the ν. Should be

$$\dot{\hat{\Phi}}_k = -\left(ik + \nu k^2\right)\hat{\Phi}_k \quad k = -N/2, \ldots, N/2.$$

   Eq. (4.38) Is also missing the ν. The RHS should be

$$-\nu \sum_{n=-N/2}^{N/2} (in)(-ik)\hat{\Phi}_n \left(e^{inx}, e^{ikx}\right)$$

8. Page 105. In eq. (4.42), the sum should be multiplied by 2π. (Thanks to David Mandel.)

9. Page 106. The second paragraph in Section 4.2.2 says "...plotted at $N_{out} = 50$ points at four times", however, the solutions are only plotted at 3 times. (Thanks to Yaning Liu.)

10. Page 108. In equation (4.51), the summation indices in the second line should be p and q, instead of n and m. (Thanks to Yaning Liu.)

11. Page 116. Alg. 50. The line $\Phi_0 = g^L(t + \Delta t)$... should be

$$\dot{\Phi}_0 = \dot{g}^L(t); \dot{\Phi}_N = \dot{g}^R(t)$$

    and moved to right after the call to *TimeDerivative*. (Thanks to Daniel Guterding)

12. Page 119. Eq. (4.82) should be

$$\varphi(x,t) = \sin[\pi(x+1)]e^{-\pi^2 t}$$

    (Thanks to Travis Johnson)

13. Page 124. Just above equation (4.91) it says $M_{kn} = (\phi_k, \phi_n), S_{kn} = (\phi_k', \phi_n')$. Although the matrices are symmetric, to be consistent with (4.90) it should be M_{nk} and S_{nk} instead. (Thanks to Yaning Liu.)

14. Page 125. Eq. (4.98) should be

$$(\varphi_{2k}, \varphi_{2n}) = \alpha_{2k}\alpha_{2n}\left\{\beta_{2k}\delta_{2k,2n} + \gamma_{2n}\delta_{2(k+1),n} + \mu_{2n}\delta_{2k,2(n+1)}\right\}$$

    (Thanks to Yaning Liu.)

15. Page 125. Eq. (4.100) should be

$$\left(\dot{\Phi}, \varphi_{2j}\right) = \beta_{2j}\alpha_{2j}^2\dot{\hat{\Phi}}_j^e + \gamma_{2j}\alpha_{2j}\alpha_{2(j-1)}\hat{\Phi}_{j-1}^e + \mu_{2j}\alpha_{2(j+1)}\alpha_{2j}\hat{\Phi}_{j+1}^e.$$

    (Thanks to Yaning Liu.)

16. Page 125. In equation (4.101), a minus sign should be added to the RHS for both equations. (Thanks to Yaning Liu.)

17. Page 126. Equation (4.103), should be

$$\begin{aligned} d_j &= \alpha_{2j+p}^2 \beta_{2j+p}, \quad j = 0, 1, \ldots, \left\lfloor \tfrac{N-2+p}{2} \right\rfloor - p \\ l_j &= \gamma_{2j+p} \alpha_{2j-p} \alpha_{2(j-1)+3p}, \quad j = 1, 2, \ldots, \left\lfloor \tfrac{N-2+p}{2} \right\rfloor - p \\ u_j &= \mu_{2j+p} \alpha_{2(j+1)+p} \alpha_{2j+p}, \quad j = 0, 1, \ldots, \left\lfloor \tfrac{N-2+p}{2} \right\rfloor - p - 1 \end{aligned}$$

    (Thanks to Yaning Liu.)

18. Page 127. Algorithm 53 should have $\Phi \leftarrow 0$ instead of $U \leftarrow 0$. (Thanks to Travis Johnson)

19. Page 133. Algorithm 57's summand in the inner loop should be $D_{k,n} D_{k,j} w_k$ to be consistent with Eq (4.123). (Thanks to Travis Johnson)

20. Page 134. In the first paragraph of 4.6.2, the second line says "revisit the example of Sect 4.1.1", however, but that example is given in Section 4.4. (Thanks to Yaning Liu.)

21. Page 134. Eq. (4.126) should have no k^2 just as (4.82). (Thanks to Yaning Liu.)

22. Page 138. After 4.141, the statement about integrate by parts once or twice is incorrect. New results show they are actually identical for either quadrature. See D.A Kopriva and G. Gassner "On the Quadrature and Weak Form Choices in Collocation Type Discontinuous Galerkin Spectral Element Methods" J. Sci. Computing. (DOI 10.1007/s10915-010-9372-3)

23. Page 139. In Alg. 60 the procedure should be called "DGDerivative", not "ComputeDgDerivative".

24. Page 140, Alg. 62, g.TimeDerivative(t) should be dg.DGTimeDerivative(t). (Thanks to James Custer.)

25. Page 141. *ComputeDGDerivative* appears in the first paragraph of 4.7.1 and the fourth line of page 141. They should be changed to *DGDerivative*. (Thanks to Yaning Liu.)

5 Chapter 5

1. Page 152. Equation (5.19): The second term should be $\frac{\partial^2 \Phi}{\partial y^2}$. (Thanks to James Custer.)

2. Page 154. Equation(5.27), for the third line, the summation index k goes from 0 to N. An $= 0$ should be added. (Thanks to Yaning Liu.)

3. Page 154. Equation(5.28), for the last line, on the LHS of the equation, the two terms should be connected by "+" instead of "-"? Also, i should go from 1 to $N-1$. (Thanks to Yaning Liu.)

4. On page 156, Alg. 66 and page 178 in Alg. 77, $\left\{ D_{i,j}^{(2),\eta} \right\}_{i,j=0}^{N}$ in the second for loop should be $\left\{ D_{i,j}^{(2),\eta} \right\}_{i,j=0}^{M}$. (Thanks to Yaning Liu and James Custer.)

5. Page 159. Equation(5.29), the second term of the first line should be D_{jk} instead of D_{ik}. (Thanks to Yaning Liu.)

6. Page 160. Algorithm 69, to find RHS_n, all D's should be changed to $D^{(2)}$, the second order derivative matrix. (Thanks to Yaning Liu.)

7. Page 162, Algorithm 71: *LaplacianOnSquare* should be *LaplacianOnTheSquare*. (Thanks to James Custer.)

8. Page 164, Eqn (5.43). In the first term the subscript on u should be $u_{i-1,j+1}$. (Thanks to Yaning Liu.)

9. Page 166, Algorithm 72. The array inputs to the constructor should start at 0. (Thanks to James Custer.)

10. Page 167, Eq. (5.52) In the third line, replace R_{ij} with R_{1j}. (Thanks to Bing Yuan.)

11. Page 169, Algorithm 75. It has been suggested to note that the auxiliary arrays should be set to zero before the iteration loop, e.g.,
$$\{p_{i,j}\}_{i,j=0}^{N,M} \leftarrow 0$$
$$\{v_{i,j}\}_{i,j=0}^{N,M} \leftarrow 0$$
(Thanks to Bing Yuan.)

12. Page 169, Algorithm 76. Line 11 should be $\rho \leftarrow 1; \alpha \leftarrow 1; \omega \leftarrow 1$ and the fifth line in the loop should be
$$\{p_{ij}\}_{i,j=0}^{N,M} \leftarrow BLAS\_SCAL(L, \beta, \{p_{ij}\}_{i,j=0}^{N,M}, 1)$$
(Thanks to Bing Yuan.)

13. Page 170, Algorithm 76. 4th line from the bottom: eta should be η. (Thanks to James Custer.)

14. Page 176. In Eqn. (5.73), should be D_{kn}. (Thanks to Yaning Liu.)

15. Page 180. Under Eqn. (5.81) should be Δx_i and Δy_j. (Thanks to Yaning Liu.)

16. Page 183. Algorithm 78. In StencilCoefficients, after the line $l = -m\ to - m + 1\ do$ add the line
$$q = (k + n) + 2 * (l + m) + 1$$
In LocalStiffnessMatrix, 5th line, replace q with p, and in line 8 replace p with q. In the next to the last line, return $\hat{S}_{kl}$. (Thanks to David Mandel.)

17. Page 183. Alg.78, Procedure StencilCoeficients In Line 6,
$$LocalStiffnessMatrix(x_{i-n+1} - x_{i-n}, y_{j-n+1} - y_{j-n})$$
should be
$$LocalStiffnessMatrix(x_{i-n+1} - x_{i-n}, y_{j-m+1} - y_{j-m})$$
(Thanks to Chaoxu Pei and Max Wise.)

18. Page 189. In Eqn (5.98), the summation indices should be $n, m = 0$ for both terms. (Thanks to Yaning Liu.)

19. Page 189. In Eqn. (5.99), all the summation indices miss the $= 0$. Also, in the first line, the first $\Phi_{n,m}$ should be $\Phi_{n,j}$ and the second $\Phi_{n,m}$ should be $\Phi_{i,m}$. (Thanks to Yaning Liu.)

20. Page 190. In Eqn. (5.110), the summation indices also miss $= 0$. Also, the first $\Phi_{n,m}$ should be $\Phi_{n,j}$ and the second $\Phi_{n,m}$ should be $\Phi_{i,m}$. (Thanks to Yaning Liu.)

21. Page 190. The first line says "the nodal values of ϕ are arbitrary and linearly dependent". Should be "linearly independent"? (Thanks to Yaning Liu.)

22. Page 192. The 1/11 factor in Eq. (5.113) should be $w_i^{(x)} w_j^{(y)}/11$. (Thanks to Yaning Liu.)

23. Page 194. Second line of the last paragraph, Algorithm 65 should be Algorithm 82. (Thanks to Yaning Liu.)

24. Page 196. Algorithm 82, in the 4th line of the main body of the algorithm, the index should be $j = 0$ to M, instead of N, for the two arrays. (Thanks to Yaning Liu.)

25. Page 197, in the 4th line, it should be Algorithm 84 (NodalAdvDiffClass:ExplicitRHS). (Thanks to Yaning Liu.)

26. Page 199. Alg. 87 needs t, N_{it}, TOL as an input.

27. Page 200. Benchmark 5.3.5 $\Delta t = 5.0 \times 10^{-3}$.

28. Page 203. Eqn(5.134) should have $k \cdot x/k - \gamma t$ in the last pair of parentheses. (Thanks to Yaning Liu.)

29. Page 204. Eqn (5.140) is missing $\hat{y}$ after $G_{n,m}$. (Thanks to Yaning Liu.)

30. Page 207. In Eqn. (5.148) in the second term on the RHS, the derivative matrix should be D^y_{kj}. (Thanks to Yaning Liu.)

31. Page 209. In eqn. (5.160) the $\vec{F}$ should be $\vec{\mathscr{F}}$. (Thanks to Andrew Winters.)

32. Page 210. Fig. 5.8: w is missing the "-" superscript on the right of the boundary. It should be as shown in Fig. 1 below.

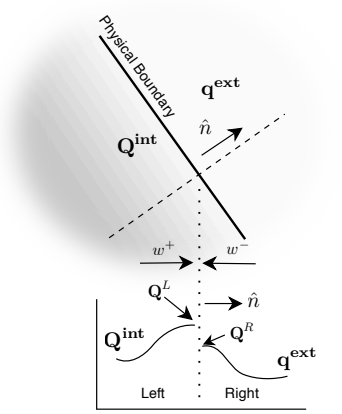

Figure 1: Interior and exterior states at a boundary viewed along the normal direction.

33. Page 213, Alg. 90. The procedure DGTimeDerivative is Alg. 93. (Thanks to James Custer.)

34. Page 213, Missing "to" before compute in the first sentence. (Thanks to James Custer.)

35. Page 214. The first line of the last paragraph should refer to 93 (DG2DTimeDerivative). (Thanks to Yaning Liu.)

36. Page 215. In Algorithm 93, for the 3rd input of the procedure SystemDGDerivative, the index $m = 1$ should be $n = 1$. The procedure SystemDGDerivative appears twice. They should both be modified. Also, for all the inputs of the second SystemDGDerivative, the superscripts N's should be M's. (Thanks to Yaning Liu.)

37. Page 218, eq. (5.171) should not have the negative sign in front of the integral. (Thanks to James Custer)

38. Page 218. Problem 5.1 should have $2\varphi_{xy}$.

39. Page 227. Algorithm 97: Two extraneous commas in the input to the constructor. (Thanks to James Custer)

40. Page 242, trucation should be truncation. (Thanks to James Custer)

6 Chapter 6

1. Page 215, Alg. 93. The first two calls to *ExternalState* are missing commas before the -1 and the 1. (Thanks to Liang Cheng.)

2. Page 234. In Eq. (6.35), the denominator in the braces should be $\Delta\xi^i\Delta\xi^j\Delta\xi^k$. (Thanks to Gonzalo Rubio.)

3. Page 241. In Eq. (6.79), the spatial derivative has been moved to the right hand side. Needs a minus sign. Should be
$$\dot{\Phi}_{i,j} = -\frac{1}{J}\left\{\frac{\partial}{\partial\xi}I_N\left[\left(Y_\eta - X_\eta\right)\Phi\right] + \frac{\partial}{\partial\eta}I_N\left[\left(-Y_\xi + X_\xi\right)\Phi\right]\right\}_{i,j}.$$

4. Page 245. In Alg. 102, $this.\eta_j$ should be $spA.\eta_j$. (Thanks to Andrew Winters.)

7 Chapter 7

1. Page 249, eq. (7.11), on the RHS the second term should be its reciprocal $1/sqrt(Y_\eta^2 + X_\eta^2)$. (Thanks to Yaning Liu.)

2. Page 249, eqn (7.12), the index should be j = 0,...M .(Thanks to Yaning Liu.)

3. Page 251, Alg.105, the indices of the input U should start from 0. (Thanks to Yaning Liu.)

4. Page 252, eqn(7.18), the indices should go to N-1 and M-1. (Thanks to Yaning Liu.) And the source term sum should have $\ell_i(\xi_n)\ell_j(\eta_m)$. (Thanks to Jonathan Panolfini.)

5. Page 253, eqn (7.21), after the second Σ on the RHS, it should be $D_{jm}^{(\eta)T}$, i. e. the subscript of D is jm. (Thanks to Yaning Liu.)

6. Page 253, eqn (7.23), after the second Σ on the RHS, it should be $D_{jm}^{(\eta)T}$, i. e. the subscript of D is jm and the superscript is ηT.(Thanks to Yaning Liu.)

7. Page 253. The + sign in front of the second sum in (7.19) should be a $-$. (Thanks to Yaning Liu.)

8. Page 255. Alg. 107. The lower index on the input U should be 0. (Thanks to Celes Woodruff.)

9. Page 256, eqn(7.24), after the second Σ on the RHS, it should be $D_{jm}^{(\eta)T}$, i. e. the subscript of D is jm and the superscript is ηT. (Thanks to Yaning Liu.)

10. Page 256, eqn (7.28), the index m should go to M. (Thanks to Yaning Liu.)

11. Page 257, eqn (7.29), the index m should go to M. (Thanks to Yaning Liu.)

12. Page 259, Alg. 108. The input to the SourceValue procedure should be x and y instead of xi and eta. (Thanks to Celes Woodruff.)

13. Page 263, Eqn. 7.42. Should have r_0^2 in the numerator. (Thanks to Yaning Liu.)

14. Page 263 eqn(7.43) , for Γ_1 and Γ_3, $\xi + 1$ should change to $1 - \xi$. (Thanks to Yaning Liu.)

15. Page 274. In the second sum of (7.78), the subscript on D should be jm. (Thanks to Pierre Garreau.)

16. Page 276, Eqn. (7.82) Curves should be:

$$\vec{\Gamma}_2(\eta) = \frac{3}{2}\hat{x} + \left(\frac{1}{2} + 0.35 * (\tanh(2) + 1)\right)\eta\hat{y}$$

and

$$\vec{\Gamma}_3(\eta) = \frac{3}{2}\xi\hat{x} + (0.3 + 0.35 * (\tanh(2\xi) + 1))\,\eta\hat{y}$$

and

$$\vec{\Gamma}_4(\eta) = -\frac{3}{2}\hat{x} + (0.3 + 0.35 * (\tanh(-2) + 1))\,\eta\hat{y}$$

(Thanks to Pierre Garreau.)

17. Page 287. Alg. 114. *this* should be replaced by dGS. (Thanks to Andrew Winters.)

18. Page 287. Alg. 114. In the calculation of G', the limits on the array $G_{j,n}$ should replace M for N. (Thanks to Andrew Winters.)

19. Page 278, Eq. (7.88) ξ and η should be replaced by $\tilde{\xi}$ and $\tilde{\eta}$ defined in (6.4).(Thanks to Pierre Garreau.)

8 Chapter 8

1. Page 299, Eq. (8.17) ϕ_x should be ϕ_ξ. (Thanks to Linlin Xu)

2. Pate 301. In (8.28), in the first line j terminates at $N-1$ and the second line should terminate at $K-1$.

3. Page 302, Alg. 116. The argument list for MatrixAction and LaplaceOperator should include $\{U_{j,k}\}_{j=0;k=1}^{N;K}$. (Thanks to Linlin Xu)

4. Page 306, Alg. 118, Procedure MatrixAction The loop on k should be to K, not $K-1$. In Procedure *Residual* in the call to MatrixAction replace $+1$ with -1. Similarly in Alg. 119, in the call to *MatrixAction* replace -1 with $+1$. Finally, before the return in *MatrixAction* add

```
IF s<0 //for the LHS
  Mask(AU)
  AU(0, 1) = 0
 AU(N, K) = 0
ENDIF
```

(Thanks to Linlin Xu)

5. Page 312, Alg. 121. *this.N* should be *this.dG.N* in the *LocalTimeDerivative* routine. (Thanks to James Custer)

6. Page 314, Alg. 122. The input to the constructor should include *nEqn* and The ExternalState routines should have time t as an input. (Thanks to James Custer)

7. Page 316. The time step should read $\Delta t = 4 \times 10^{-3}$.

8. Page 324. Alg. 125. In the constructor, *.nodeIDs* should be *.nodes*. (Thanks to Andrew Winters.)

9. Page 327. Alg. 127. $.elementID_2$ should be $.elementIDs_2$. (Thanks to Andrew Winters.)

10. Page 330-331. Eqns. (8.69)-(8.72). All instances of Φ should have superscripts k.

11. Page 333. In Alg. 129, the next to the last line should read $mesh.Construct(this.spA, meshFile)$.

12. Page 338. In Alg. 132, $sum \leftarrow tmp_{j,1} + tmpn, 2$. (Thanks to Pierre Garreau.)
13. Page 346. Alg. 138. $this.elements_k$ should be $this.mesh.elements_k$. (Thanks to Andrew Winters.)
14. Page 349, Eq. (8.82) In the second line, the ρ_L should be ρ_R. (Thanks to Andrew Winters.)
15. Page 349, Eq. (8.86) In the first row, the nu should be $n_x u$. (Thanks to Andrew winters and Liang Cheng.)

9 Appendix E

1. Page 380. In Alg. 147, Procedure "GetDataForKeys" should be "DataForKeys".