Section 3.8 - Steps in Solving Polynomial and Rational Inequalities

STEP 1: Write the inequality so that a polynomial or rational expression \( f \) is on the left side and zero is on the right side in one of the following forms:

\[
\begin{align*}
f(x) &> 0 \\
f(x) &\geq 0 \\
f(x) &< 0 \\
f(x) &\leq 0
\end{align*}
\]

For rational expressions, be sure that the left side is written as a single quotient. This step converts the problem of solving an inequality into an equivalent (i.e., the same solution) problem of determining where a function is positive (or negative).

STEP 2: Factor \( f(x) \) to determine the numbers at which the expression \( f(x) \) on the left side equals zero and, if the expression is rational, the numbers at which the expression \( f \) on the left side is undefined. We will call these numbers partition values.

The Intermediate Value Theorem tells us that a continuous function (graph can be drawn without raising the pencil from the paper) cannot change signs on an interval without having a zero in that interval. So the above partition values divide the \( x \)-axis into intervals on which the sign of \( f(x) \) CANNOT change.

STEP 3: Use the numbers found in STEP 2 to separate the real number line into intervals.

We will construct a sign chart for the function \( f(x) \).

STEP 4: Determine the sign of \( f(x) \) on the intervals found in step 3.

STEP 5: The solution of the inequality includes all intervals with the correct sign (positive or negative).

If the inequality is not strict, include the numbers at which \( f(x) \) is zero in the solution set.

Be careful: The numbers which make \( f(x) \) undefined (i.e., the zeroes of the denominator in a rational function) are never included in the solution set.