Test to determine if a function y=f(x) is even, odd or neither:

Replace x with -x and compare the result to f(x).

If $f(-x) = f(x)$, the function is even.

If $f(-x) = -f(x)$, the function is odd.

If $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$, the function is neither even nor odd.

Remember that $(-x)^n = \begin{cases} x^n, & \text{if n is even} \\ -x^n, & \text{if n is odd} \end{cases}$

So, taking (-x) to an **even** power gives:

$(-x)^2 = (-x)(-x) = x^2$, $(-x)^4 = (-x)(-x)(-x)(-x) = x^4$, $(-x)^6 = (-x)(-x)(-x)(-x)(-x)(-x) = x^6$

Taking (-x) to an **odd** power gives:

$(-x)^3 =(-x)(-x)(-x) = - x^3$, $(-x)^5 = (-x)(-x)(-x)(-x)(-x) = - x^5$, etc.

Terms which involve **odd** powers of x will **change signs** when x is replaced with (-x).

Terms which involve **even** powers of x will **remain the same** when x is replaced with (-x).

And since **constant** terms do not involve x, they will also **remain the same** when x is replaced with (-x).

<u>Examples</u>:

a) *Functions whose terms contain only EVEN powers of the variable x and possibly a constant term (but no terms containing ODD powers of x) are likely to be EVEN functions. For example:*

$f(x) = x^4 - 3x^2 + 7$

Replacing x with -x we obtain: $f(-x) = (-x)^4 - 3(-x)^2 + 7 = x^4 - 3x^2 + 7 = f(x)$.

So f(x) is **even**.

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b) *Functions which contain a term with an EVEN power of x* <u>*and*</u> *a term with an ODD power of x or, at least one term with an ODD power of x* <u>*and*</u> *a constant term are likely to be NEITHER even nor odd. . For example:*

$f(x) = x^5 - 3x^3 + 7$

Replacing x with -x we obtain: $f(-x) = (-x)^5 - 3(-x)^3 + 7 = -x^5 + 3x^3 + 7$.

Notice that the first two terms in f(x) changed signs when x was replaced with (-x), but the third (constant) term did not .
So the resulting f(-x) is not equal to f(x) or -f(x). And f(x) is **neither** even nor odd.

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c) *Functions which contain only terms with ODD powers of x (no terms with EVEN powers of x and no CONSTANT terms) are likely to be ODD.*

Also, <u>***rational functions***</u> <u>***whose numerator is an ODD function***</u> ***and*** <u>***denominator is an EVEN function***</u> ***(or vice versa) are likely to be*** <u>***ODD functions***</u>***.*** *. For example:*

$$f(x) = \frac{x^2 + 4}{x^3 - x}.$$

Replacing x with -x we obtain:

$$f(-x) = \frac{(-x)^2 + 4}{(-x)^3 - (-x)} = \frac{x^2 + 4}{-x^3 + x} = \frac{x^2 + 4}{-(x^3 - x)} = -\frac{x^2 + 4}{x^3 - x} = -f(x)$$

So f(x) is odd.