Chapter 12. Vectors and the Geometry of Space

12.1 Three-Dimensional Coordinate System

12.1.1 3D Space

Recall:

- To represent points in space :
  - Choose a fixed point O (the origin)
  - Coordianate axes : x, y and z.

Definition 1

1. **Coordinates**

2. **Cooridate planes**, for example xy-plane, xz-plane, $\cdots$

3. **Octants**

4. The point $P(a, b, c)$ determines a rectangular box. If we drop a perpendicualr from P to the xy-plane, we get a point Q with coordinates $(a, b, 0)$ called the _____ of P onto the xy-plane.

5. The Cartesian product $\mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{(x, y, z) | x, y, z \in \mathbb{R}\}$ is the set of all ordered triples of real numbers and is denoted by $\mathbb{R}^3$.

12.1.2 Surfaces

Recall:

- The graph of an equation $f(x, y)$ involving x and y in $\mathbb{R}^2$ is a line/curve.
- In $\mathbb{R}^3$, an equation $f(x, y, z)$ in x, y, and z represents a **surface**.

Example 1. What surfaces in $\mathbb{R}^3$ are represented by the following equations?

(a) $z = 3$ (b) $y = 5$

Example 2. (b) What does the equation $x^2 + y^2 = 1$ represent as a surface in $\mathbb{R}^3$?

(a) Which points (x, y, z) satisfy the equations

$$x^2 + y^2 = 1 \quad \text{and} \quad z = 3$$

12.1.3 Distance and Spheres

Definition 2 Distance Formula in Three Dimensions. The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$|P_1P_2| := \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + \underline{(z_2 - z_1)^2}}$$

Example 4. The distance from the point $P(2, -1, 7)$ to the point $Q(1, -3, 5)$ is _____.

Example 5. Find an equation of a sphere with radius r and center $C(a, b, c)$?
Recall: How about in two dimension? an equation of a circle?

Definition 3 Equation of a Sphere, in particular, if the center is the origin $O(0, 0, 0)$:

Example 6. Show that $x^2 + y^2 + z^2 + 4x - 6y + 2z + 6 = 0$ is the equation of a sphere, and find its center and radius.
Recall: factorization, perfect squares

Example 7. What region in $\mathbb{R}^3$ is represented by the following inequalities?

$$1 \leq x^2 + y^2 + z^2 \leq 4, \quad z \leq 0$$

Suggested Home Work Problems: 12.1 Exercises (8th edition)

$$\# : 13, 15, 17 - 20, 47$$