Chapter 12. Vectors and the Geometry of Space

12.2 Vectors

Definition 1 The term **vector** has both __________ and __________.

Notations:

12.2.1 Combining Vectors

Definition 2 Definition of Vector Addition: If $\mathbf{u}+\mathbf{v}$ are vectors positioned so the initail point of $\mathbf{v}$ is at the terminal point of $\mathbf{u}$, then the sum $\mathbf{u}+\mathbf{v}$ is the vecotr from the initial point of $\mathbf{u}$ to the terminal point of $\mathbf{v}$.

Example 1. Draw the sum of the vectors $\mathbf{a}$ and $\mathbf{b}$.

Definition 3 Definition of Scalar Multiplication: If c is a scalar and $\mathbf{v}$ is a vector, then the scalar multiple $c\mathbf{v}$ is the vector whose length is $|c|$ times the length of $\mathbf{v}$ and whose directions is the _____ as $\mathbf{v}$ if $c > 0$ and is _____ to $\mathbf{v}$ if $c < 0$. If $c = 0$ or $\mathbf{v} = 0$ then $c\mathbf{v} =$ ___.

Definition 4 Definition of Vector Subtraction (Difference):

$$\mathbf{u}-\mathbf{v}=\mathbf{u}+(-\mathbf{v})$$

Example 2. Draw $\mathbf{a} - 2\mathbf{b}$

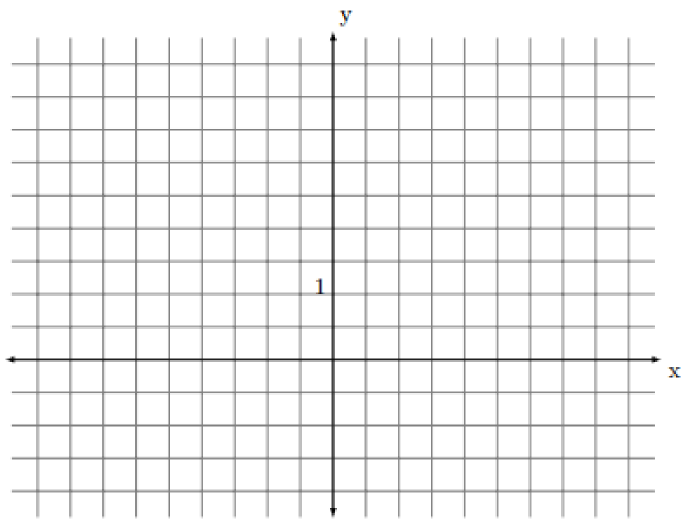
12.2.2 Components

Definition 5 If we place the initial point of a vector $\mathbf{a}$ at the origin of a rectangular coordinate system, then the terminal point of $\mathbf{a}$ has coordinates of the form $< a_1, a_2 >$ or $< a_1, a_2, a_3 >$. These coordinates are called the _______ of $\mathbf{a}$ and we write _______

Definition 6 Given the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, the vector $\mathbf{a}$ with representation $\overrightarrow{AB}$ is

$$\mathbf{a} =< x_2 - x_1, y_2 - y_1, z_2 - z_1 >$$

Definition 7 The _______ of the vector $\mathbf{a} =< 3, 2 >$ are...



Example 3. Find the vector represented by the directed line segment with initial point $A(2, -3, 4)$ and terminal point $B(-2, 1, 1)$.

Definition 8 The particular _______ of the vector $\mathbf{u} = \overrightarrow{OP}$ from the origin to the point $P(a, b)$ is called the ______________ of the point P.

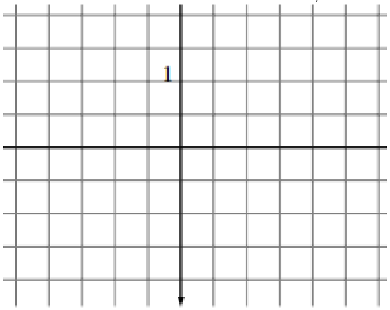
Definition 9 The _______ or _______ of the vector **v** is the length of any of its representations and is denoted by the symbol

_______ or _______

The length (magnitude) of the three-dimensional vector $\mathbf{a} =< a_1, a_2, a_3 >$ is

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

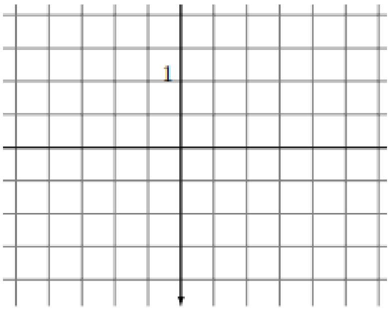
Remark. How do we add/subtract vectors algebraically?



If $\mathbf{a} =< a_1, a_2, a_3 >$, $\mathbf{b} =< b_1, b_2, b_3 >$, and c is a scalar constant then

1. $\mathbf{a} + \mathbf{b} =$
2. $\mathbf{a} - \mathbf{b} =$
3. $c\mathbf{a} =$

Example 4. If $\mathbf{a} =< 4, 0, 3 >$ and $\mathbf{b} =< -2, 1, 5 >$, find $|\mathbf{a}|$ and the vectors $\mathbf{a} + \mathbf{b}$, $\mathbf{a} - \mathbf{b}$, $3\mathbf{a}$, and $2\mathbf{a} + 5\mathbf{b}$. Can we link these concepts to the previous geometric vectors?



Properties of Vectors. If $\mathbf{a}$,$\mathbf{b}$ and $\mathbf{c}$ are vectors in V_n and c and d are scalars, then

1. $\mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a}$
2. $\mathbf{a}+(\mathbf{b}+\mathbf{c})=(\mathbf{a}+\mathbf{b})+\mathbf{c}$
3. $\mathbf{a}+\mathbf{0}=$ _______
4. $\mathbf{a}-\mathbf{a}=$ _______
5. $\mathbf{c}(\mathbf{a}+\mathbf{b})=$ _______
6. $(c+d)\mathbf{a}=$ _______
7. $(cd)\mathbf{a}=c(d\mathbf{a})$
8. $1\mathbf{a}=\mathbf{a}$

Definition 10 Standard basis vectors: $\mathbf{i},\mathbf{j}$ and $\mathbf{k}$:

$$\mathbf{i}=<1,0,0>,\ \mathbf{j}=<0,1,0>,\ \mathbf{k}=<0,0,1>$$

If $\mathbf{a}=<a_1,a_2,a_3>$ then we can write:

Example 5. If $\mathbf{a}=\mathbf{i}+2\mathbf{j}$ - $3\mathbf{k}$ and $\mathbf{b}=4\mathbf{i}+7\mathbf{k}$, express the vector $2\mathbf{a}+3\mathbf{b}$ in terms of $\mathbf{i},\mathbf{j}$, and $\mathbf{k}$.

Definition 11 A __________ is a vector whose length is __. If a vector $\mathbf{a} \neq 0$ then the unit vector that has the same direction as $\mathbf{a}$ is

$$\mathbf{u} = \frac{1}{|\mathbf{a}|}\mathbf{a} = \frac{\mathbf{a}}{|\mathbf{a}|}$$

Example 6. Find the unit vector in the direction of the vector $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$

Example 7. A 100-lb weight hangs from two wires as shown in Figure 19. Find the tensions (forces) $\mathbf{T}_1$ and $\mathbf{T}_2$ in both wires and the magnitudes of the tensions.

Suggested Home Work Problems: 12.2 Exercises (8th edition)

$$\#15-18, 19-22, 23-25, 26, 37, 41$$