Chapter 12. Vectors and the Geometry of Space

* How about multiplications between vectors?

12.3 The Dot Product

Definition 1 If $\mathbf{a} =< a_1, a_2, a_3 >$ and $\mathbf{b} =< b_1, b_2, b_3 >$ then the _____________ of $\mathbf{a}$ and $\mathbf{b}$ is the number $\mathbf{a} \cdot \mathbf{b}$ given by

$$\mathbf{a} \cdot \mathbf{b} = \_\_\_\_\_\_\_\_\_\_\_\_\_$$

The result is not a _______, but it is a _______.

Example 1.

1. $< 2, 4 > \cdot < 3, -1 >=$
2. $< -1, 7, 4 > \cdot < 6, 2, -0.5 >=$
3. $(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) \cdot (2\mathbf{j} - \mathbf{k}) =$

Properties

1. $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$
2. $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$
3. $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$
4. $(c\mathbf{a}) \cdot \mathbf{b} = c(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot (c\mathbf{b})$
5. $0 \cdot \mathbf{a} = 0$

* What does the dot product really mean? Why do we need this?

___________ between the vectors

Theorem 2 If θ is the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$, then

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$$

Example 2. If the vectors $\mathbf{a}$ and $\mathbf{b}$ have lengths 4 and 6, then the angle between them is $\pi/3$, find $\mathbf{a} \cdot \mathbf{b}$

Corollary 6. If θ is the angle between the nonzero vectors $\mathbf{a}$ and $\mathbf{b}$, then

$$\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

Example 3. Find the angle between the vectors $\mathbf{a} =< 2, 2, -1 >$ and $\mathbf{b} =< 5, -3, 2 >$.

Definition 3 Two nonzero vectors $\mathbf{a}$ and $\mathbf{b}$ are called **perpendicular** and **orthogonal** if the angle between them is $\theta =$ _ _ _ _ _. Then $\mathbf{a}\cdot\mathbf{b} =$ _ _ _ _ _ _ _ _ _ _ _ _ _ _.

Theorem 4 Two vectors $\mathbf{a}$ and $\mathbf{b}$ are orthogonal if and only if $\mathbf{a}\cdot\mathbf{b} = 0$.

Example 4. Are these two vectors $2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $5\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$ perpendicular?

* $\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|$, if two vectors : _ _ _ _ _ _ _ _ _ _ _ _.

* $\mathbf{a}\cdot\mathbf{b} = -|\mathbf{a}||\mathbf{b}|$, if two vectors : _ _ _ _ _ _ _ _ _ _ _ _.

12.3.1 Direction Angles and Direction Cosines

Definition 5 The ___________________ of a nonzero vector **a** are the angels $\alpha, \beta, \gamma \in [0, \pi]$ that **a** makes with the positive $x-, y-$ and $z-$ axes. The cosines of these direction angles, $\cos\alpha, \cos\beta, \cos\gamma$ are called the _____________________ of the vectore **a**.

Remark 6 Using Corollary 6 with **b** replaced by **i**, we obtain

$$\cos\alpha = \frac{\mathbf{a}\cdot\mathbf{i}}{|\mathbf{a}||\mathbf{i}|} = \frac{a_1}{|\mathbf{a}|}$$

Similarly, we also have

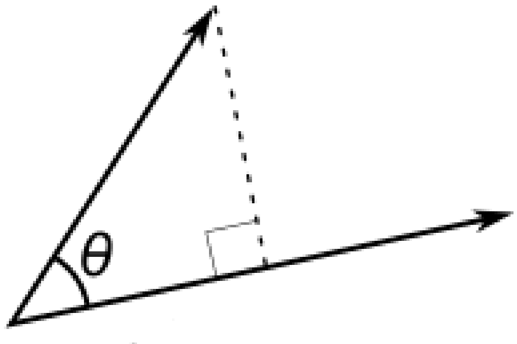
$$\cos\beta = \qquad\qquad\qquad \cos\gamma =$$

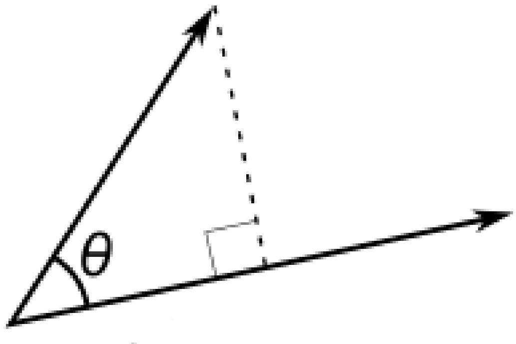
Then, we get

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma =$$

Example 5. Find the direction angles of the vector $\mathbf{a} = < 1, 2, 3 >$.

12.3.2 Projections





Definition 7 Projections:

- Scalar projection of $\mathbf{b}$ onto $\mathbf{a}$: $\text{comp}_{\mathbf{a}}\mathbf{b} := \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$

- Vector projection of $\mathbf{b}$ onto $\mathbf{a}$: $\text{proj}_{\mathbf{a}}\mathbf{b} := \left(\dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}\right)\dfrac{\mathbf{a}}{|\mathbf{a}|} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\mathbf{a}$

Example 6. Find the scalar projections and vector projection of $\mathbf{b} =< 1, 1, 2 >$ onto $\mathbf{a} =< -2, 3, 1 >$.

Suggested Home Work Problems: 12.3 Exercises (8th edition)

$$\# : 15 - 20, 23 - 24, 39 - 44$$