

## CHAPTER 12. VECTORS AND THE GEOMETRY OF SPACE

### 12.1 THREE-DIMENSIONAL COORDINATE SYSTEM

#### 12.1.1 3D Space

Recall:

- To represent points in space :
  - Choose a fixed point  $O$  (the origin)
  - Coordinate axes :  $x, y$  and  $z$ .

**Definition 1**

1. **Coordinates**

2. **Coordinate planes**, for example  $xy$ -plane,  $xz$ -plane,  $\dots$

3. **Octants**

4. The point  $P(a, b, c)$  determines a rectangular box. If we drop a perpendicular from  $P$  to the  $xy$ -plane, we get a point  $Q$  with coordinates  $(a, b, 0)$  called the \_\_\_\_\_ of  $P$  onto the  $xy$ -plane.

5. The Cartesian product  $\mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{(x, y, z) | x, y, z \in \mathbb{R}\}$  is the set of all ordered triples of real numbers and is denoted by  $\mathbb{R}^3$ .

## 12.1.2 Surfaces

**Recall:**

- The graph of an equation  $f(x, y)$  involving  $x$  and  $y$  in  $\mathbb{R}^2$  is a line/curve.
- In  $\mathbb{R}^3$ , an equation  $f(x, y, z)$  in  $x, y$ , and  $z$  represents a **surface**.

**Example 1.** What surfaces in  $\mathbb{R}^3$  are represented by the following equations?

(a)  $z = 3$

(b)  $y = 5$

**Example 2.** (b) What does the equation  $x^2 + y^2 = 1$  represent as a surface in  $\mathbb{R}^3$  ?

(a) Which points  $(x, y, z)$  satisfy the equations

$$x^2 + y^2 = 1 \quad \text{and} \quad z = 3$$

### 12.1.3 Distance and Spheres

**Definition 2** Distance Formula in Three Dimensions. The distance  $|P_1P_2|$  between the points  $P_1(x_1, y_1, z_1)$  and  $P_2(x_2, y_2, z_2)$  is

$$|P_1P_2| := \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

**Example 4.** The distance from the point  $P(2, -1, 7)$  to the point  $Q(1, -3, 5)$  is \_\_\_\_\_.

**Example 5.** Find an equation of a sphere with radius  $r$  and center  $C(a, b, c)$ ?

**Recall:** How about in two dimension? an equation of a circle?

**Definition 3** Equation of a Sphere, in particular, if the center is the origin  $O(0, 0, 0)$ :

**Example 6.** Show that  $x^2 + y^2 + z^2 + 4x - 6y + 2z + 6 = 0$  is the equation of a sphere, and find its center and radius.

Recall: factorization, perfect squares

**Example 7.** What region in  $\mathbb{R}^3$  is represented by the following inequalities?

$$1 \leq x^2 + y^2 + z^2 \leq 4, \quad z \leq 0$$

**Suggested Home Work Problems: 12.1 Exercises (8th edition)**

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