Let C_1 be $x^2+y^2 = ax$, for which $2x + 2yy' = a$

$$\Rightarrow \quad y'_{C_1} = \frac{a-2x}{2y}$$ 

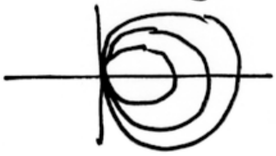
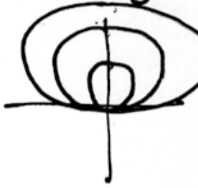
Let C_2 be $x^2+y^2 = by$, for which $2x+2yy' = by'$

$$\Rightarrow \quad y'_{C_2} = \frac{-2x}{2y-b}$$ 

C_1 meets C_2 where $x^2+y^2 = ax$ AND $x^2+y^2 = by$

$$\Rightarrow \quad y = \frac{ax}{b} \quad \text{and} \quad x^2 + \left(\frac{ax}{b}\right)^2 = ax$$

$$\Rightarrow \quad x^2\left[1 + \frac{a^2}{b^2}\right] = ax \qquad (*)$$

Let's assume that neither x nor y is zero, because we already know that these circles  and these circles 

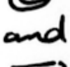
cross orthogonally at the origin (because the coordinate axes are the tangents, and they're orthogonal). Then $x \neq 0$ in $(*)$ yields

$$x\left[1 + \frac{a^2}{b^2}\right] = a \quad \Rightarrow \quad x = \frac{ab^2}{a^2+b^2}$$

$$\Rightarrow \quad y = \frac{ax}{b} = \frac{a^2b}{a^2+b^2}$$

So, at the point of intersection (other than the origin) we have

$$y'_{C_1}\, y'_{C_2} = \frac{a - \frac{2ab^2}{a^2+b^2}}{\frac{2a^2b}{a^2+b^2}} \cdot \frac{\left\{-\frac{2ab^2}{a^2+b^2}\right\}}{\frac{2a^2b}{a^2+b^2} - b}$$

(from  and )

$$= \frac{a(a^2+b^2)-2ab^2}{2a^2b}\;\frac{\{-2ab^2\}}{2a^2b - b(a^2+b^2)}$$

$$= \frac{a(a^2-b^2)}{2a^2b} \cdot \frac{\{-2ab^2\}}{b(a^2-b^2)} = -1$$