Set $f(x) = \dfrac{\sqrt{x+2} - 3}{x-7}$

Then, because $\lim\limits_{x \to 7} \sqrt{x+2} - 3 = \sqrt{7+2} - 3 = \sqrt{9} - 3 = 3 - 3 = 0$

and $\lim\limits_{x \to 7} x - 7 = 7 - 7 = 0$,

$f(7)$ is undefined (it is indeterminate of type $\frac{0}{0}$)

but we can still find $\lim\limits_{x \to 7} f(x)$ if we multiply cleverly by 1:

Write $f(x) = f(x) \cdot 1 = f(x) \cdot \dfrac{\sqrt{x+2}+3}{\sqrt{x+2}+3}$

$$= \frac{(\sqrt{x+2} - 3)(\sqrt{x+2} + 3)}{(x-7)(\sqrt{x+2} + 3)}$$

$$= \frac{(\sqrt{x+2})^2 - 3^2}{(x-7)(\sqrt{x+2} + 3)}$$

$$= \frac{x+2 - 9}{(x-7)(\sqrt{x+2} + 3)}$$

$$= \frac{x-7}{(x-7)(\sqrt{x+2} + 3)}$$

$$= \frac{1}{\sqrt{x+2} + 3} \quad \text{whenever } x \neq 7$$

(which is good enough for finding the limit). So now

$$\lim_{x \to 7} f(x) = \lim_{x \to 7} \frac{1}{\sqrt{x+2} + 3}$$

$$= \frac{1}{\sqrt{7+2} + 3} = \frac{1}{\sqrt{9}+3}$$

$$= \frac{1}{3+3} = \frac{1}{6}$$