Use the quotient rule in conjunction with

$$\frac{d}{du}\left[e^{au}\right] = ae^{au} \qquad (*)$$

for any constant a. (The proof of $(*)$ is as follows: put $w = au$, $z = e^{w}$. Then $\frac{d}{du}\left[e^{au}\right] = \frac{d}{du}\left[e^{w}\right] = \frac{dz}{du} = \frac{dz}{dw}\frac{dw}{du} = e^{w}\cdot a = ae^{au}$ by the chain rule.) With $a = -1$ we have $\frac{d}{du}(e^{-u}) = (-1)e^{-u} = -e^{-u}$; with $a = 2$ we have $\frac{d}{du}(e^{2u}) = 2e^{2u}$. So

$$\frac{d}{du}\left(\frac{e^{2u}}{e^{u}+e^{-u}}\right) = \frac{\frac{d}{du}(e^{2u})\cdot(e^{u}+e^{-u}) - e^{2u}\frac{d}{du}(e^{u}+e^{-u})}{(e^{u}+e^{-u})^2}$$

$$= \frac{2e^{2u}(e^{u}+e^{-u}) - e^{2u}\{e^{u}-e^{-u}\}}{(e^{u}+e^{-u})^2}$$

$$= \frac{e^{2u}}{(e^{u}+e^{-u})^2}\left\{2e^{u}+2e^{-u} - e^{u} + e^{-u}\right\}$$

$$= \frac{e^{2u}}{(e^{u}+e^{-u})^2}\left\{e^{u} + 3e^{-u}\right\} = \frac{e^{u}(e^{2u}+3)}{(e^{u}+e^{-u})^2}$$

if you prefer. Or if you don't like that either, write the answer as

$$\frac{e^{3u}(e^{2u}+3)}{(e^{2u}+1)^2}$$