For this kind of problem, all you can do is guess, then verify.

We have

$$f(x) = (2-x)^{-1}.$$

So, by the chain rule, and with $u = 2-x \Rightarrow \frac{du}{dx} = 0-1$,

$$f'(x) = \frac{d}{du}[u^{-1}]\frac{du}{dx} = (-1)u^{-2}\cdot(-1)$$

$$= (2-x)^{-2}$$

$$f''(x) = \frac{d}{dx}\left[f'(x)\right] = \frac{d}{dx}(u^{-2})$$

$$= \frac{d}{du}(u^{-2})\frac{du}{dx}$$

$$= -2u^{-3}(-1)$$

$$= 2\cdot(2-x)^{-3}$$

$$f'''(x) = \frac{d}{dx}\left[f''(x)\right] = \frac{d}{dx}(2u^{-3})$$

$$= 2\frac{d}{du}(u^{-3})\frac{du}{dx}$$

$$= 2(-3)u^{-4}(-1)$$

$$= 2\cdot 3\cdot(2-x)^{-4}$$

and similarly

$$f^{(4)}(x) = 2\cdot 3\cdot 4\,(2-x)^{-5}$$

$$f^{(5)}(x) = 2\cdot 3\cdot 4\cdot 5\,(2-x)^{-6}, \text{ etc.}$$

So it appears that

$$f^{(n)}(x) = 1.2.3.4.5.\ \dots\ n\,(2-x)^{-(n+1)}$$

$$= \frac{n!}{(2-x)^{n+1}} \qquad (☺)$$

and you can verify by mathematical induction (see p. 81) that this guess is correct (in essence, show that (☺) $\Rightarrow$

$$f^{(n+1)}(x) = \frac{(n+1)!}{(2-x)^{(n+1)+1}} = \frac{(n+1)!}{(2-x)^{(n+2)}}\ )$$