1 (a) \[ \frac{3^2 - 9}{3^2 + 6 - 3} = 0 \] (because rational function continuous where denominator \( \neq 0 \))

(b) \[ \frac{x^2 - 9}{x^2 + 2x - 3} = \frac{(x+3)(x-3)}{(x+3)(x-1)} = \frac{x-3}{x-1} \quad \text{for all } x \neq 3 \]

\[ \lim_{x \to -3} \frac{x^2 - 9}{x^2 + 2x - 3} = \lim_{x \to -3} \frac{-2}{-1} = \frac{2}{1} = 2 \]

(c) \[ +\infty \] (because near \( x = 1 \), numerator \( \approx -2 \) and denominator small and \( < 0 \))

(d) \[ -\infty \] (because near \( x = -2 \), denominator \( \approx 0 \) and \( > 0 \))

(e) \[ 1 \]

\[ \lim_{x \to \infty} \frac{1 - \frac{a^2}{x}}{1 + 2/x - 3/x^2} = \frac{1}{1} = 1 \]

(f) \[ 1 \]

\[ \lim_{x \to -\infty} \frac{1}{1 + 2/x - 3/x^2} = \frac{1}{1} = 1 \]

(g) \[ \frac{f}{g} \]

\[ \lim_{x \to 1} \frac{f}{g} = +\infty \quad \text{and} \quad \lim_{x \to 1} \frac{g}{f} = -\infty \]

(h) \[ \lim_{x \to 2} \frac{\sqrt{x+2} - \sqrt{2x}}{x^2 - 2x} = \lim_{x \to 2} \frac{\sqrt{x+2} - \sqrt{2x}}{x(x-2)} \cdot \frac{\sqrt{x+2} + \sqrt{2x}}{\sqrt{x+2} + \sqrt{2x}} \]

\[ = \lim_{x \to 2} \frac{x+2 - 2x}{x(x-2)(\sqrt{x+2} + \sqrt{2x})} \quad \text{(because } (a-b)(a+b) = a^2 - b^2 \text{ with } a = \sqrt{x+2}, b = \sqrt{2x}) \]

\[ = \lim_{x \to 2} \frac{2-x}{(x-2)(\sqrt{x+2} + \sqrt{2x})} = \lim_{x \to 2} \frac{-1}{x(\sqrt{x+2} + \sqrt{2x})} = \frac{-1}{2(\sqrt{4} + \sqrt{4})} = \frac{-1}{8} \]

2 (a) \[ f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a} \]

\[ = \lim_{x \to a} \frac{6}{\sqrt{x+2} - \sqrt{a+2}} \]

\[ = \lim_{x \to a} \frac{6}{x-a} \left\{ \frac{1}{\sqrt{x+2}} - \frac{1}{\sqrt{a+2}} \right\} \]

\[ = \lim_{x \to a} \frac{6}{x-a} \left\{ \frac{\sqrt{a+2} - \sqrt{x+2}}{\sqrt{a+2} \sqrt{x+2}} \right\} \]

\[ = \lim_{x \to a} \frac{6}{x-a} \left\{ \frac{(a+2) - (x+2)}{x-a} \right\} \]

\[ = \lim_{x \to a} \frac{6(a-x)}{x-a} \left\{ \sqrt{x+2} \sqrt{a+2} \right\} \]

\[ = \lim_{x \to a} \frac{6(a-x)}{(x-a)(\sqrt{x+2} \sqrt{a+2})} \]

\[ = \lim_{x \to a} \frac{-6}{\sqrt{x+2} \sqrt{a+2}} \]

\[ = \frac{-6}{\sqrt{a+2} \sqrt{a+2}} = \frac{-3}{(a+2)^{3/2}} \]

\[ \Rightarrow f'(x) = \frac{-3}{(x+2)^{3/2}} \]

Both \( f \) and \( f' \) have domain \((-2, \infty)\)

Domain: \((2, \infty)\) for both \( f \) & \( f' \)
3 (a) Set \( f(x) = x^3 - 3x + 2 \) \( \Rightarrow \) \( f'(x) = 3x^2 - 3 + 0 \) \( \Rightarrow \) \( f'(i) = 0 \)
and \( g(x) = e^x + 4x^2 + 1 \) \( \Rightarrow \) \( g'(x) = e^x + 4 \cdot 2x + 0 \)
\( \Rightarrow \) \( g'(i) = e + 8 \)
Then \( h(x) = \frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x) \)
\( \Rightarrow \) \( h'(i) = f'(i)g(i) + f(i)g'(i) = 0 + (1^3 - 3+2)(e+8) \)
\( \Rightarrow \) a = \( \frac{e+5}{(e+5)^2} \)
(But the method would be essentially the same even if the answers were not zero.)
(b) \( h(x) = \frac{f(x)}{g(x)} \) \( \Rightarrow \) \( h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2} \)
\( \Rightarrow \) \( h'(i) = \frac{g(i)f'(i) - f(i)g'(i)}{g(i)^2} = \frac{(e+5) \cdot 0 - 0 \cdot (e+8)}{(e+5)^2} = 0 \)
(Same comment.)
4. \( f'(x) = \begin{cases} 
2bx & \text{if } x < 2 \\
-1 & \text{if } x > 2
\end{cases} \) (by linearity and power law)
Continuity of \( f \) requires \( f(-2) = f(2) \) or \( a \cdot 2 + b \cdot 2^2 = \frac{1}{2} \)
" " " \( f' \) " " \( f'(-2) = f'(2) \) or \( a + 2b \cdot 2 = -\frac{1}{2} \)
So \( 2a + 4b = \frac{1}{2} \)
\( \Rightarrow \) \( a = \frac{3}{4}, b = -\frac{1}{4} \)
5. By the quotient rule, \( \frac{dy}{dx} = \frac{(x+1) \frac{d}{dx}(x) - x \frac{d}{dx}(x+1)}{(x+1)^2} = \frac{(x+1) \cdot 1 - x \cdot 1}{(x+1)^2} = \frac{1}{(x+1)^2} \)
So tangent line at \( (a, \frac{a}{a+1}) \) has slope \( \frac{1}{(a+1)^2} \)
So its equation is \( y - \frac{a}{a+1} = \frac{1}{(a+1)^2} \left\{ x - a \right\} \)
It passes through \( (1, 2) \) if \( 2 \cdot \frac{a}{a+1} = \frac{1-a}{(a+1)^2} \)
\( 2(a+1)^2 - a(a+1) = 1 - a \) or \( a^2 + 4a + 1 = 0 \) \( \Rightarrow \) \( (a+2)^2 = 3 \)
\( \Rightarrow \) \( a = -2 \pm \sqrt{3} \). So there are two such tangent lines. They touch the curve at \( \left( -2 \pm \sqrt{3}, \frac{-2 \pm \sqrt{3}}{-2 \pm \sqrt{3} + 1} \right) \), or \( \left( 3-2, \frac{\sqrt{3}-2}{\sqrt{3}+1} \right) \) and \( \left( -3-2, \frac{-\sqrt{3}}{\sqrt{3}+1} \right) \)