

**Mock Third Test**  
 Wednesday, July 14, 2004

**You are allowed to use a TI-30Xa (or any four-function calculator). No other calculator is allowed. You have one hour. Present your solutions clearly. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are stapled together in the proper order at the end of the test. You may assume that**

$$\frac{d}{d\xi} \{\arctan(\xi)\} = \frac{1}{\xi^2 + 1}.$$

1. Given that  $f(0) = 20$ ,  $f'(4) = 7$  and  $f''(t) = \frac{3}{\sqrt{t}}$  for all  $t > 0$ , find  $f(t)$  *exactly*. [6]

2. (a) Calculate  $\frac{d}{dx} \{\arctan(\sqrt{2x^2 + 1})\}$ . [2]

(b) Find the *exact* value of  $\int_0^1 \left\{ \frac{5}{x+1} + \frac{3x}{(x^2+1)\sqrt{2x^2+1}} \right\} dx$ . [4]

3. In each of the following cases, find the exact value of the definite integral:

(a)  $\int_4^9 \left\{ \sqrt{x} + \frac{2}{\sqrt{x}} \right\}^2 dx$  [5]    (b)  $\int_0^{\pi/4} |\cos(3x)| dx$  [6]    (c)  $\int_0^7 (4x+5)(3x-2) dx$  [5]

4. Find the area of the region bounded above by the line  $y = 1$ , to the left by the line  $x = 0$  and to the right and below by the graph  $y = x^{1/5}$ . [6]

5. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other piece is bent into a circle. How should the wire be cut so that the total area enclosed is

- (a) a maximum?
- (b) a minimum? [8]

6. A hungry dog at point  $H$  on the edge of a circular pond spots a bone at the diametrically opposite point  $B$ . If the dog swims half as fast as it runs and wants to reach the bone as soon as possible, how should it proceed? [8]