1. \( f''(t) = 3t^{-1/2} = \frac{d}{dt} [6t^{1/2}] \Rightarrow f'(t) = 6t^{1/2} + B, \) where \( f'(4) = 7 \Rightarrow 7 = 6 \sqrt{4} + B \Rightarrow B = 7 - 12 = -5. \) So \( f'(t) = 6t^{1/2} - 5 = \frac{d}{dt} \left\{ 4t^{3/2} - 5t \right\} \Rightarrow f(t) = 4t^{3/2} - 5t + C. \) But \( f(0) = 20 \Rightarrow 20 = 4 \cdot 0 - 0 + C \Rightarrow C = 20. \) So \( f(t) = 4t^{3/2} - 5t + 20. \)

2(a) Put \( u = (2x^2+1)^{1/2} \) and use the Chain Rule to get 
\[
\frac{d}{dx} \left[ \arctan(u) \right] = \frac{1}{u^2 + 1} \frac{du}{dx} = \frac{1}{2x^2 + 1} \frac{1}{2} (2x^2 + 1)^{-1/2} \frac{d}{dx} (2x^2 + 1) \]
\[
= \frac{1}{2x^2 + 1} \cdot \frac{1}{2} \cdot (4x + 0) = \frac{x}{2x^2 + 1}.
\]

(b) \[
\int_0^1 (5 + \frac{3x}{(2x^2+1)^{1/2}}) \, dx = 5 \int_0^1 \frac{1}{x+1} \, dx + 3 \int_0^1 \frac{3x}{(2x^2+1)^{1/2}} \, dx = 5 \ln(x+1) \bigg|_0^1 + 3 \left\{ \frac{\pi}{3} - \frac{\pi}{4} \right\} = 5 \ln(2) + \frac{\pi}{12}.
\]

3(a) \[
\int_0^9 \left( \sqrt{x} + \frac{2}{\sqrt{x}} \right)^2 \, dx = \int_0^9 \left( x + \frac{4}{x} + 2 \cdot \sqrt{x} \cdot \frac{2}{\sqrt{x}} \right) \, dx = \int_0^9 \left( x + \frac{4}{x} + 4 \right) \, dx.
\]

\[
= \int_0^9 \left\{ \frac{1}{2} x^2 + 4 \ln(x) + 4x \right\} \, dx = \left\{ \frac{1}{2} x^2 + 4 \ln(x) + 4x \right\} \bigg|_0^9 = \frac{1}{2} (9^2 - 0^2) + 4 \ln(9) - 4 \ln(0) + 4 \cdot 9 = \frac{1}{2} (81 + 4 \ln(9)) + 4 \ln(9) + 36 = \frac{1}{2} \cdot 5 + 4 \ln \left[ \frac{3}{2} \right]^2 + 20 = \frac{105}{2} + 4 \cdot 2 \ln \left( \frac{3}{2} \right) = \frac{105}{2} + 8 \ln \left( \frac{3}{2} \right) = \frac{105}{2} + 8 \ln \left( \frac{3}{2} \right).
\]

(b) Let \( \cos(3x) \) such that \( \cos(3x) \geq 0 \) and \( 0 \leq x \leq \frac{\pi}{4} \), i.e., where \( 0 \leq 3x \leq \frac{\pi}{2} \) or \( 0 \leq x \leq \frac{\pi}{6} \). Otherwise, i.e., if \( \cos(3x) < 0 \), \( [\cos(3x)] = -\cos(3x) \). So \( [\cos(3x)] = -\cos(3x) \) where \( \pi/6 < x < \pi/4 \).

\[
\int_0^{\pi/6} \left| \cos(3x) \right| \, dx = \int_0^{\pi/6} \cos(3x) \, dx + \int_{\pi/6}^{\pi/4} \left\{ -\cos(3x) \right\} \, dx.
\]

\[
= \int_0^{\pi/6} \frac{d}{dx} \left[ \frac{1}{3} \sin(3x) \right] \, dx - \int_{\pi/6}^{\pi/4} \frac{d}{dx} \left[ \frac{1}{3} \sin(3x) \right] \, dx = \frac{1}{3} \sin(3x) \bigg|_0^{\pi/6} - \frac{1}{3} \sin(3x) \bigg|_{\pi/6}^{\pi/4} = \frac{2}{3} \sin \left( \frac{\pi}{2} \right) - 0 - \frac{1}{3} \sin \left( \frac{3\pi}{4} \right) = \frac{2}{3} - \frac{1}{3} \cdot \frac{1}{\sqrt{2}} = \frac{4 - \sqrt{2}}{6}.
\]
5. Let the square have side $x$. Then the circle has circumference $10-4x$. If its radius is $r$ then $2\pi r = 10-4x$

$$\Rightarrow r = \frac{5-2x}{\pi}$$

So total enclosed area is $A = x^2 + \frac{(5-2x)^2}{\pi}$, which we want to extremize for $0 \leq x \leq \frac{5}{2}$. We have

$$\frac{dA}{dx} = \frac{2x}{\pi} + \frac{2}{\pi} \left(5-2x\right) \frac{d}{dx} \left(5-2x\right) = 2x + \frac{2}{\pi} \left(5-2x\right)(-2) = 2 \left(1 + \frac{4}{\pi}\right) x - \frac{10}{\pi}$$

Also $\frac{d^2A}{dx^2} = 2 \left(1 + \frac{4}{\pi}\right) > 0$. So we have a minimum where $\left(1 + \frac{4}{\pi}\right)x = \frac{10}{\pi}$ and a maximum at one of the endpoints. Because $A = \frac{25}{\pi}$ when $x = 0$ and $A = \frac{25}{4}$ when $x = \frac{5}{2}$, the maximum clearly occurs where $x = 0$. So

(a) Don't cut it, bend it into a circle and get maximum area $\frac{25}{\pi}$.

(b) Make a square of side $\frac{10}{\pi + 4}$ and a circle of radius $\frac{5}{\pi + 4}$ to achieve the minimum area $\left(\frac{10}{\pi + 4}\right)^2 + \pi \left(\frac{5}{\pi + 4}\right)^2 = \frac{25}{\pi + 4}$.

6. Suppose the dog contemplates swimming from $H$ to $S$ and running from $S$ to $B$. Let $v$ be its running velocity, and let $r$ be the radius of the pond. Then $\frac{1}{2}v$ is the dog's swimming velocity. Let $\angle OHS = \theta$. Then $\angle SOB = 2\angle OHS = 2\theta$, $\angle HSBO = \frac{\pi}{2}$ and $v$ and $r$ are both constant. So

$$T = \text{travel time} = \frac{SH}{\frac{1}{2}v} + \frac{\text{arc } SB}{\frac{1}{2}v} = \frac{HB \cos \theta + \text{radius} \cdot 2\theta}{\frac{1}{2}v} = \frac{2r \cos \theta + r \cdot 2\theta}{\frac{1}{2}v}$$

and

$$\frac{dT}{d\theta} = \frac{-4r \cos \theta}{v} < 0 \text{ for } 0 < \theta < \frac{\pi}{2}. \text{ So the interior critical point where } \sin \theta = \frac{1}{2} \text{ or } \theta = \frac{\pi}{6} \text{ is a maximum. But we want the minimum on } [0, \frac{\pi}{2}], \text{ which clearly occurs when } \theta = \frac{\pi}{2} \text{ because then}$$

$$T = \frac{2r}{v}, \text{ whereas } T = \frac{4r}{v} \text{ when } \theta = 0. \text{ So the dog should run the whole way, and enjoy a dry bone.}$$