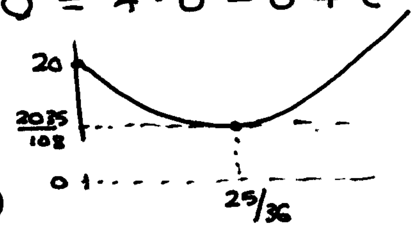
1. $f''(t) = 3t^{-1/2} = \frac{d}{dt}\left[6t^{1/2}\right] \Rightarrow f'(t) = 6t^{1/2} + B$, where $f'(4) = 7 \Rightarrow 7 = 6\sqrt{4} + B \Rightarrow B = 7 - 12 = -5$. So $f'(t) = 6t^{1/2} - 5 = \frac{d}{dt}\left\{4t^{3/2} - 5t\right\}$ $\Rightarrow f(t) = 4t^{3/2} - 5t + C$. But $f(0) = 20 \Rightarrow 20 = 4 \cdot 0 - 0 + C$ $\Rightarrow C = 20$. So $f(t) = \underline{\underline{4t\sqrt{t} - 5t + 20}}$.



2(a) Put $u = (2x^2+1)^{1/2}$ and use the Chain Rule to get

$$\frac{d}{dx}\left\{\arctan(u)\right\} = \frac{1}{u^2+1}\frac{du}{dx} = \frac{1}{2x^2+1+1}\,\frac{1}{2}(2x^2+1)^{-1/2}\cdot\frac{d}{dx}(2x^2+1)$$

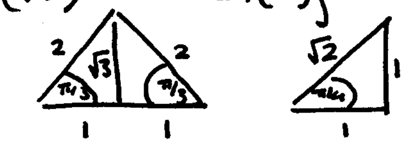
$$= \frac{1}{2x^2+2}\cdot\frac{1}{2\sqrt{2x^2+1}}\cdot(4x+0) = \underline{\underline{\frac{x}{(x^2+1)\sqrt{2x^2+1}}}}.$$

(b) $$\int_0^1\left(\frac{5}{x+1} + \frac{3x}{(x^2+1)\sqrt{2x^2+1}}\right)dx = 5\int_0^1\frac{1}{x+1}\,dx + 3\int_0^1\frac{x}{(x^2+1)\sqrt{2x^2+1}}\,dx =$$

$$5\int_0^1\frac{d}{dx}\left[\ln(x+1)\right]dx + 3\int_0^1\frac{d}{dx}\left\{\arctan\left(\sqrt{2x^2+1}\right)\right\}dx = 5\ln(x+1)\Big|_0^1 +$$

$$3\arctan\left(\sqrt{2x^2+1}\right)\Big|_0^1 = 5\left\{\ln(2) - \ln(1)\right\} + 3\left\{\arctan(\sqrt{3}) - \arctan(1)\right\} =$$

$$5\left\{\ln(2) - 0\right\} + 3\left\{\frac{\pi}{3} - \frac{\pi}{4}\right\} = \underline{\underline{5\ln(2) + \frac{1}{4}\pi}}$$



3 (a) $$\int_4^9\left(\sqrt{x} + \frac{2}{\sqrt{x}}\right)^2dx = \int_4^9\left(x + \frac{4}{x} + 2\cdot\sqrt{x}\cdot\frac{2}{\sqrt{x}}\right)dx = \int_4^9\left(x + \frac{4}{x} + 4\right)dx$$

on using $(A+B)^2 = A^2 + B^2 + 2AB$

$$= \int_4^9\frac{d}{dx}\left\{\frac{1}{2}x^2 + 4\ln(x) + 4x\right\}dx = \left\{\frac{1}{2}x^2 + 4\ln(x) + 4x\right\}\Big|_4^9$$

$$= \frac{1}{2}9^2 + 4\ln(9) + 4\cdot 9 - \left\{\frac{1}{2}\cdot 4^2 + 4\ln(4) + 4\cdot 4\right\}$$

$$= \frac{1}{2}(9^2 - 4^2) + 4\left\{\ln(9) - \ln(4)\right\} + 4(9-4) = \frac{1}{2}(9-4)(9+4) + 4\ln\left(\frac{9}{4}\right) + 4\cdot 5$$

$$= \frac{1}{2}\cdot 5\cdot 13 + 4\ln\left[\left(\frac{3}{2}\right)^2\right] + 20 = \frac{105}{2} + 4\cdot 2\ln\left(\frac{3}{2}\right) = \underline{\underline{\frac{105}{2} + 8\ln\left(\frac{3}{2}\right)}}$$

(b) $|\cos(3x)| = \cos(3x)$ where $\cos(3x) \geq 0$ and $0 \leq x \leq \pi/4$, i.e., where $0 \leq 3x \leq \pi/2$ or $0 \leq x \leq \pi/6$. Otherwise, i.e., if $\cos(3x) < 0$, $|\cos(3x)| = -\cos(3x)$. So $|\cos(3x)| = -\cos(3x)$ where $\pi/6 < x \leq \pi/4$.

$$\therefore \int_0^{\pi/4}|\cos(3x)|\,dx = \int_0^{\pi/6}\cos(3x)\,dx + \int_{\pi/6}^{\pi/4}\left\{-\cos(3x)\right\}dx$$

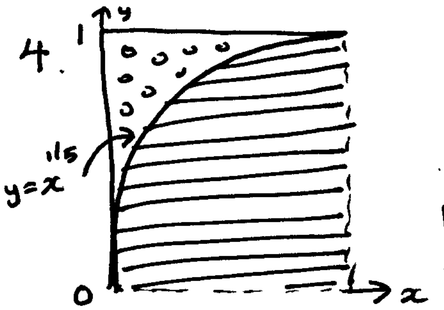
$$= \int_0^{\pi/6}\frac{d}{dx}\left[\frac{1}{3}\sin(3x)\right]dx - \int_{\pi/6}^{\pi/4}\frac{d}{dx}\left[\frac{1}{3}\sin(3x)\right]dx = \frac{1}{3}\sin(3x)\Big|_0^{\pi/6} -$$

$$\frac{1}{3}\sin(3x)\Big|_{\pi/6}^{\pi/4} = \frac{1}{3}\sin\left(\frac{\pi}{2}\right) - \frac{1}{3}\sin(0) - \left\{\frac{1}{3}\sin\left(\frac{3\pi}{4}\right) - \frac{1}{3}\sin\left(\frac{\pi}{2}\right)\right\}$$

$$= \frac{2}{3}\sin\left(\frac{\pi}{2}\right) - 0 - \frac{1}{3}\sin\left(\frac{3\pi}{4}\right) = \frac{2}{3}\cdot 1 - \frac{1}{3}\cdot\frac{1}{\sqrt{2}}$$

$$= \underline{\underline{\frac{4 - \sqrt{2}}{6}}}$$

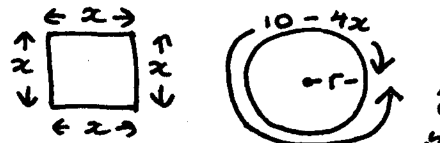
4.



$\text{ooo} = \underset{\text{area}}{\text{square}} - \equiv = 1^2 - \int_0^1 x^{1/5}\,dx = 1 - \int_0^1 \frac{d}{dx}\left(\frac{5}{6}x^{6/5}\right)dx$

$= 1 - \frac{5}{6}x^{6/5}\Big|_0^1 = 1 - \frac{5}{6}\left\{1^{6/5} - 0^{6/5}\right\} = 1 - \frac{5}{6} = \underline{\underline{\frac{1}{6}}}$

5.



Let the square have side x. Then the circle has circumference $10-4x$. If its radius is r then $2\pi r = 10-4x$ $\Rightarrow r = \dfrac{5-2x}{\pi}$ $\Rightarrow \text{area} = \pi r^2 = \dfrac{(5-2x)^2}{\pi}$

So total enclosed area is $A = x^2 + \dfrac{(5-2x)^2}{\pi}$, which we want to extremize for $0 \le x \le 5/2$. We have $\dfrac{dA}{dx} = \dfrac{d}{dx}(x^2) + \dfrac{1}{\pi}\dfrac{d}{dx}\left\{(5-2x)^2\right\}$

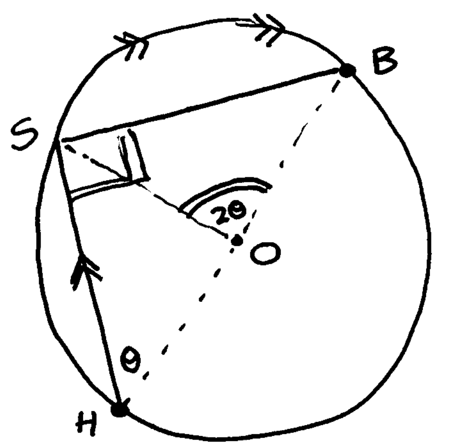
$= 2x + \frac{1}{\pi}\,2(5-2x)\frac{d}{dx}(5-2x) = 2x + \frac{2}{\pi}(5-2x)(-2) = 2\left\{\left(1+\frac{4}{\pi}\right)x - \frac{10}{\pi}\right\}$

Also $\dfrac{d^2A}{dx^2} = 2\left(1+\dfrac{4}{\pi}\right) > 0$. So we have a minimum where $\left(1+\dfrac{4}{\pi}\right)x = \dfrac{10}{\pi}$ and a maximum at one of the endpoints. Because $A = \dfrac{25}{\pi}$ when $x=0$ and $A = \dfrac{25}{4}$ when $x = \dfrac{5}{2}$, the maximum clearly occurs where $x = 0$. So

(a) Don't cut it, bend it into a circle and get maximum area $25/\pi$.

(b) Make a square of side $\dfrac{10}{\pi+4}$ and a circle of radius $\dfrac{5}{\pi+4}$ to achieve the minimum area $\left(\dfrac{10}{\pi+4}\right)^2 + \pi\left(\dfrac{5}{\pi+4}\right)^2 = \dfrac{25}{\pi+4}$.

6.



Suppose the dog contemplates swimming from H to S and running from S to B. Let v be its running velocity, and let r be the radius of the pond. Then $\frac{1}{2}v$ is the dog's swimming velocity. Let $\angle OHS = \theta$. Then $\angle SOB = 2\angle OHS = 2\theta$, $\angle HSB = \pi/2$ and v and r are both constant. So

$T = \text{travel time} = \dfrac{SH}{\frac{1}{2}v} + \dfrac{\text{arc } SB}{v} = \dfrac{HB\cos\theta}{\frac{1}{2}v} + \dfrac{\text{radius}\cdot 2\theta}{v}$

$= \dfrac{2r\cos\theta}{\frac{1}{2}v} + \dfrac{r\cdot 2\theta}{v} = \dfrac{2r}{v}\left\{2\cos\theta + \theta\right\} \Rightarrow \dfrac{dT}{d\theta} = \dfrac{2r}{v}\left\{-2\sin\theta + 1\right\}$

and $\dfrac{d^2T}{d\theta^2} = -\dfrac{4r\cos\theta}{v} < 0$ for $0 < \theta < \dfrac{\pi}{2}$. So the interior critical point where $\sin\theta = 1/2$ or $\theta = \pi/6$ is a maximum. But we want the minimum on $[0, \pi/2]$, which clearly occurs where $\theta = \pi/2$ because then $T = \pi r/v$, whereas $T = \dfrac{4r}{v}$ when $\theta = 0$. So the dog should run the whole way, and enjoy a dry bone.