1. $\dfrac{dy}{dx} = (0+2x)(2+x^2)^{1/3} + (1+x^2)\,\dfrac{1}{3}(2+x^2)^{-2/3}(0+2x)$ [BY THE PRODUCT & CHAIN RULES]

$\Rightarrow \left.\dfrac{dy}{dx}\right|_{x=5} = 10\cdot 27^{1/3} + \dfrac{26}{3}(27)^{-2/3}\cdot 10 = \underline{\underline{\dfrac{1070}{27}}}$

2. Write $f(x) = \dfrac{1}{\ln(x)} - \dfrac{1}{x-1} = \dfrac{x-1-\ln(x)}{(x-1)\ln(x)}$. Because this has form $\frac{0}{0}$

L'Hôpital yields $\displaystyle\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{\frac{d}{dx}\{x-1-\ln(x)\}}{\frac{d}{dx}[(x-1)\ln(x)]} = \lim_{x\to 1} \frac{1-0-\frac{1}{x}}{(1-0)\ln(x) + (x-1)\cdot\frac{1}{x}}$.

$\boxed{\text{BECAUSE } \ln(1)=0}$

This still has form $\frac{0}{0}$, so apply L'Hôpital again: $\displaystyle\lim_{x\to 1} f(x) = \lim_{x\to 1}\frac{\frac{d}{dx}(1-1/x)}{\frac{d}{dx}\left(\ln(x)+1-\frac{1}{x}\right)}$

$\displaystyle = \lim_{x\to 1}\frac{0+\frac{1}{x^2}}{1/x + 0 + \frac{1}{x^2}} = \frac{0+\frac{1}{1^2}}{1/1 + 0 + 1/1^2} = \frac{1}{2}$.

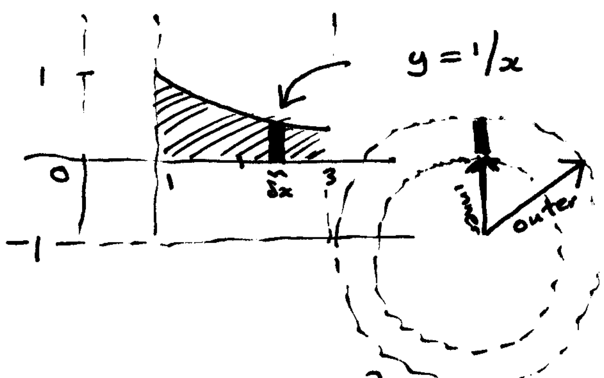
3. $u = (1+2x)^{1/3} \Rightarrow u^3 = 1+2x \Rightarrow x = \frac{1}{2}u^3 - \frac{1}{2} \Rightarrow \dfrac{dx}{du} = \dfrac{3}{2}u^2$. So

$\displaystyle I = \int_{x=0}^{x=13}\left(\frac{1}{20}+x^2\right)\frac{1}{(1+2x)^{1/3}}\,dx = \int_{u=(1+0)^{1/3}}^{u=(1+26)^{1/3}}\left(\frac{1}{20}+x^2\right)\frac{1}{(1+2x)^{1/3}}\frac{dx}{du}\,du =$

$\displaystyle \int_1^3\left\{\frac{1}{20} + \frac{(u^3-1)^2}{4}\right\}\frac{1}{u}\,\frac{3}{2}u^2\,du = \frac{3}{8}\int_1^3\left\{\frac{1}{5} + (u^3-1)^2\right\}u\,du = \frac{3}{8}\int_1^3\left\{u^7 - 2u^4 + \frac{6u}{5}\right\}du$

$\displaystyle = \frac{3}{8}\left(\frac{u^8}{8} - \frac{2u^5}{5} + \frac{3u^2}{5}\right)\Big|_1^3 = \frac{3}{8}\left\{\frac{3^8}{8} - \frac{2\cdot 3^5}{5} + \frac{3^3}{5} - \frac{1}{8} + \frac{2}{5} - \frac{3}{5}\right\} = \underline{\underline{273}}$

4. (a)



$\delta V \approx \pi\{\text{outer}^2 - \text{inner}^2\}\delta x$

$= \pi\left(\left\{\frac{1}{x} - (-1)\right\}^2 - \{0-(-1)\}^2\right)\delta x$

$\displaystyle\Rightarrow V = \int_{x=1}^{x=3}\pi\left\{\left(\frac{1}{x}+1\right)^2 - 1\right\}dx$

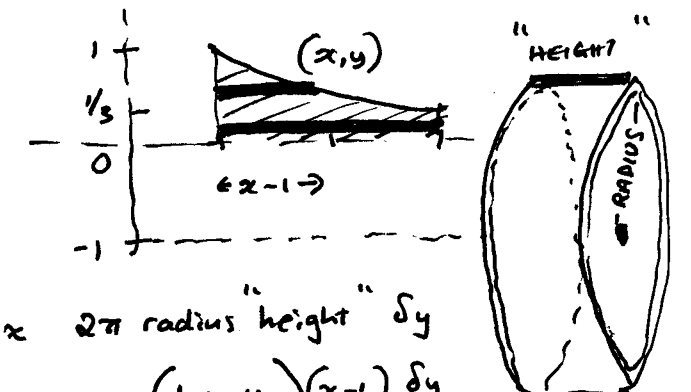
$\displaystyle = \pi\int_1^3\left\{x^{-2} + 2x^{-1}\right\}dx$

$= \pi\left(-x^{-1} + 2\ln(x)\right)\Big|_1^3$

$= \pi\left\{-\frac{1}{3} + 2\ln(3) - (-1) - 2\ln(1)\right\}$

$= 2\pi\left(\ln(3) + \frac{1}{3}\right)$

(b)



$\delta V \approx 2\pi$ radius "height" δy

$= 2\pi(1+y)(x-1)\,\delta y$ if $1/3 \le y \le 1$

but $= 2\pi(1+y)(3-1)\,\delta y$ if $0 \le y \le 1/3$

So $\displaystyle V = \int_{y=0}^{y=1/3} 2\pi(1+y)\cdot 2\,dy + \int_{y=1/3}^{y=1} 2\pi(1+y)\left(\frac{1}{y}-1\right)dy$

$\displaystyle = 4\pi\int_0^{1/3}(1+y)\,dy + 2\pi\int_{1/3}^1\left(\frac{1}{y} - y\right)dy$

$\displaystyle = 4\pi\,\frac{(1+y)^2}{2}\Big|_0^{1/3} + 2\pi\left(\ln(y) - \frac{1}{2}y^2\right)\Big|_{1/3}^1$

$= \frac{4\pi}{2}\left\{\left(\frac{4}{3}\right)^2 - 1^2\right\} + 2\pi\left\{\ln(1) - \frac{1}{2} - \ln\left(\frac{1}{3}\right) + \frac{1}{18}\right\}$

$= 2\pi\left\{\frac{16}{9} - 1 + 0 - \frac{1}{2} + \ln(3) + \frac{1}{18}\right\}$

$= 2\pi\left(\ln(3) + 1/3\right)$