Note this correction to the key on the following page:

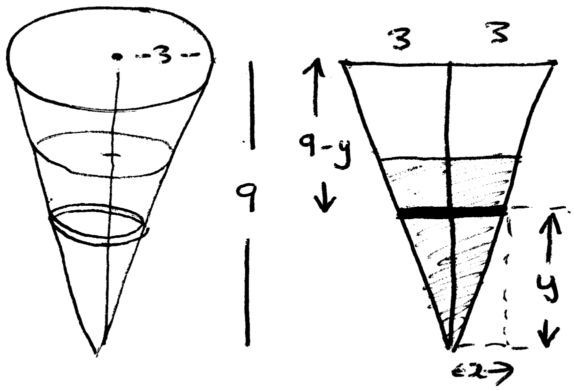
L: http://www.math.fsu.edu/˜mesterto/Courses/MAC2311/2004/Tests/20040804a.pdf

1. See A, #3
2. See A, #5
3. See B, #1
4. See B, #2
5. See B, #3
6. See B, #4
7. See B, #5
8. See C, #3
9. See C, #4
10. See C, #5
11. See D, #5
12. See E, #2
13. **(a)-(b)** See E, #3. **(c)** See D, #4.
14. See F, #1
15. See F, #2
16. See F, #3
17. See F, #4
18. See F, #5
19. See G, #1
20. See G, #2
21. See H, #3
22. See H, #1
23. See H, #2
24. See I, #5
25. **(a)** See J, #4. **(b)** See overleaf.
26. See K, #2
27. See L, #4
28. See overleaf.

Key:

A: http://www.math.fsu.edu/˜mesterto/Courses/MAC2311/2004/Tests/20040916a.pdf (first test solutions)
B: http://www.math.fsu.edu/˜mesterto/Courses/MAC2311/2004/Tests/20041005a.pdf (second test solutions)
C: http://www.math.fsu.edu/˜mesterto/Courses/MAC2311/2004/Tests/20041026a.pdf (third test solutions)
D: http://www.math.fsu.edu/˜mesterto/Courses/MAC2311/2004/Tests/20041118a.pdf (fourth test solutions)
E: http://www.math.fsu.edu/˜mesterto/Courses/MAC2311/2004/Tests/20041112a.pdf (mock fourth test solutions)
F: http://www.math.fsu.edu/˜mesterto/Courses/1999/Spring/MAC2311/990420.html
G: http://www.math.fsu.edu/˜mesterto/Courses/1999/Fall/MAC2311/991201.html
H: http://www.math.fsu.edu/˜mesterto/Courses/1999/Fall/MAC2311/991216.html
I: http://www.math.fsu.edu/˜mesterto/Courses/MAC2311/2004/Supplementary/20040527a.pdf
J: http://www.math.fsu.edu/˜mesterto/Courses/MAC2311/2004/Tests/20040623a.pdf
K: http://www.math.fsu.edu/˜mesterto/Courses/MAC2311/2004/Tests/20040714a.pdf
L: http://www.math.fsu.edu/˜mesterto/Courses/MAC2311/2004/Tests/20040714a.pdf

25(b)



Let ρ = density (mass/volume) of water.

By similar Δs, $\frac{y}{x} = \frac{9}{3} = 3 \Rightarrow x = y/3$. So $\delta V = \pi x^2 \delta y + o(\delta y)$

$$= \pi (y/3)^2 \delta y + o(\delta y)$$

$$\Rightarrow \delta W = \underbrace{\underbrace{\rho\,\delta V}_{\text{MASS}}\ \underbrace{g}_{\text{GRAVITY}}}_{\text{FORCE}}\ \underbrace{\{9 - y + o(\delta y)\}}_{\text{DISTANCE}}$$

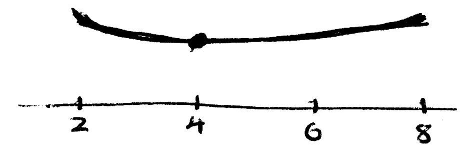
$$= \rho g \pi (y/3)^2 (9-y)\,\delta y + o(\delta y) \Rightarrow$$

$$W = \int_{y=0}^{y=6} \rho g \pi \left(\frac{y}{3}\right)^2 (9-y)\,dy = \frac{1}{9}\rho g \pi \int_0^6 (9y^2 - y^3)\,dy =$$

$$\frac{1}{9}\rho g \pi \left(3y^3 - \frac{1}{4}y^4\right)\Big|_0^6 = \frac{1}{9}\rho g \pi \left(3\cdot 6^3 - \frac{1}{4}6^4 - 0\right) = 36\rho g \pi$$

ergs (CGS units) $= \frac{36\rho g \pi}{10^7}$ joules (SI units)

28(a)



$$f(x) = \ln(7) + \ln(x) + \frac{4}{x}$$

$$\Rightarrow f'(x) = 0 + \frac{1}{x} - \frac{4}{x^2}$$

$$\Rightarrow f''(x) = \frac{-1}{x^2} + \frac{8}{x^3} = \frac{8-x}{x^3} > 0 \text{ for } 2 \le x < 8.$$

So unique global minimum where $f'(x) = 0$ or $x = 4$ $(0 \in [2,8])$

Also $f(2) = \ln(14) + 2 > f(8) = \ln(56) + \frac{1}{2}$. Hence

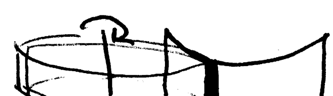
The global max is $f(2) = \ln(14) + 2$ and the global min is

$f(4) = \ln(28) + 1$

(b) $$\int_2^8 f(x)\,dx = \int_2^8 \left(\ln(7) + \ln(x) + \frac{4}{x}\right) dx = \left(\ln(7)x + x\ln(x) - x + 4\ln(x)\right)\Big|_2^8$$

$$= 8\ln(7) + 8\ln(8) - 8 + 4\ln(8) - \ln(7)\cdot 2 - 2\ln(2) + 2 - 4\ln(2)$$

$$= 6\ln(7) + 30\ln(2) - 6 \quad (\approx 26.47)$$



(c) $$\delta V = 2\pi r h\, \delta t + o(\delta t)$$

$$= 2\pi x f(x)\, \delta x + o(\delta x) \Rightarrow$$

$$V = \int_2^8 2\pi x f(x)\,dx = 2\pi \int_2^8 \left(\ln(7)x + x\ln(x) + 4\right) dx$$

$$= 2\pi \left\{ \frac{1}{2}\ln(7)x^2 + \frac{1}{4}x^2\{\ln(x^2) - 1\} + 4x \right\}\Big|_2^8$$

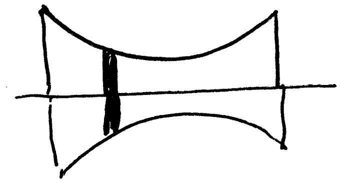
$$= 2\pi \left\{ \frac{1}{2}\ln(7)\cdot 8^2 + \frac{1}{4}8^2[\ln(8^2) - 1] + 4\cdot 8 - \frac{1}{2}\ln(7)\cdot 2^2 - \frac{1}{4}2^2[\ln(2^2) - 1] - 4\cdot 2 \right\}$$

$$= 2\pi\{30\ln(7) + 94\ln(2) + 9\} \quad (\approx 132.5)$$

(d)

$$\delta V = \pi \text{ radius}^2 \, \delta t + o(\delta t)$$

$$= \pi \{f(x)\}^2 \, \delta x + o(\delta x) \quad \Rightarrow$$



$$V = \int_2^8 \pi [f(x)]^2 \, dx$$

$$= \pi \int_2^8 \left\{ \ln(7) + \ln(x) + \frac{4}{x} \right\}^2 dx$$

$$= \pi \int_2^8 \left\{ \ln(7)^2 + \ln(x)^2 + \frac{16}{x^2} + 2\ln(7)\ln(x) + \frac{8\ln(7)}{x} + \frac{8\ln(x)}{x} \right\} dx$$

$$= \pi \ln(7)^2 \int_2^8 1 \, dx + \pi \int_2^8 \ln(x)^2 \, dx + 16\pi \int_2^8 x^{-2} \, dx$$

$$+ 2\pi \ln(7) \int_2^8 \ln(x) \, dx + 8\pi \ln(7) \int_2^8 \frac{1}{x} \, dx + 8\pi \int_2^8 \frac{\ln(x)}{x} \, dx$$

$$= \pi \ln(7)^2 (8-2) + \pi x\{(\ln(x))^2 - \ln(x^2) + 2\}\Big|_2^8 - \frac{16\pi}{x}\Big|_2^8$$

$$+ 2\pi \ln(7) \left(x\ln(x) - x\right)\Big|_2^8 + 8\pi \ln(7) \ln(x)\Big|_2^8 + 4\pi \ln(x)^2 \Big|_2^8$$

$$= 6\pi \ln(7)^2 + 8\pi\{(\ln(8))^2 - \ln(64) + 2\} - 2\pi\{(\ln(2))^2 - \ln(4) + 2\}$$

$$- 2\pi + 8\pi + 2\pi \ln(7)\{8\ln(8) - 8 - 2\ln(2) + 2\}$$

$$+ 8\pi \ln(7)\{\ln(8) - \ln(2)\} + 4\pi\{[\ln(8)]^2 - [\ln(2)]^2\}$$

$$= 12\pi\{\ln(8)\}^2 + 6\pi\{\ln(7)\}^2 - 6\pi\{\ln(2)\}^2 + 18\pi$$

$$+ 2\pi\{-4\ln(64) + \ln(4) - 6\ln(7)\}$$

$$+ 24\pi \ln(7)\ln(8) - 12\pi \ln(7)\ln(2)$$

$$= 6\pi\left\{2[\ln(8)]^2 + [\ln(7)]^2 - [\ln(2)]^2 + 3 + 10\ln(7)\ln(2)\right\}$$

$$- 44\pi \ln(2) - 12\pi \ln(7) \approx 366.95$$