MAC 2311, Section 02, Summer 2004 (Dr M-G)

Second Test

Wednesday, June 23, 2004

You are allowed to use a TI-30Xa (or any four-function calculator). No other calculator is allowed. You have one hour. Present your solutions clearly. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are stapled together in the proper order at the end of the test.

You may assume that the volume of a cone of radius r and height h is $\frac{1}{3}\pi r^2 h$.

1. In each of the following cases, calculate $\left.\dfrac{dy}{dx}\right|_{x=a}$ *exactly* for the given value of a:

(a) $y = e^{3x}\sin(5x)$, $a = \frac{\pi}{4}$ [6]

(b) $y = \dfrac{(x+1)^2}{\sqrt{2x^2+1}}$, $a = 2$ [6]

(c) $y = x\,e^{1/x}$, $a = -\frac{1}{2}$ [6]

(d) $y = \ln(x)^{x+2\sqrt{x}}$, $a = e$ [6]

2. **(a)** What is the tangent line at $(-1,5)$ to the hyperbola $x^2+4xy+y^2=6$? [5]

(b) Find $\left.\dfrac{d^2y}{dx^2}\right|_{(x,y)=(-1,5)}$ (for the same hyperbola). [6]

Note (to check result): By a curious coincidence, 2(b) and 1(b) have the same answer.

(c) Is the hyperbola concave up or concave down at $(-1,5)$? [1]

3. Find the absolute maximum and minimum values of $f(x) = 2x^3 - 3x^2 - 12x + 9$ on $[-3,3]$, together with all corresponding maximizers and minimizers. [8]

4. A paper cup has the shape of an inverted cone with height 9 cm and radius 3 cm. If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 6 cm deep? [6]

[Perfect score: $24 + 12 + 8 + 6 = 50$]