1 (a) $y' = 3e^{3x}\cdot\sin(5x) + e^{3x}\cdot 5\cos(5x) \Rightarrow \left.\frac{dy}{dx}\right|_{x=\pi/4} = e^{3\pi/4}\left\{3\sin\left(\frac{5\pi}{4}\right) + 5\cos\left(\frac{5\pi}{4}\right)\right\}$

$= e^{3\pi/4}\left\{3\left(\frac{-1}{\sqrt2}\right) + 5\left(\frac{-1}{\sqrt2}\right)\right\} = -\frac{8}{\sqrt2}e^{3\pi/4} = \underline{\underline{-4\sqrt2\, e^{3\pi/4}}}$

(b) $y = (x+1)^2(1+2x^2)^{-1/2} \Rightarrow \ln(y) = 2\ln(x+1) - \frac12\ln(1+2x^2) \Rightarrow \frac1y\frac{dy}{dx}$

$= 2\cdot\frac{1}{x+1}(1+0) - \frac12\cdot\frac{1}{1+2x^2}(0+4x) = 2\left\{\frac{1}{x+1} - \frac{x}{1+2x^2}\right\}$. When $x=2$, $y = \frac{(2+1)^2}{\sqrt{2\cdot2^2+1}}$

$= \frac{3^2}{\sqrt9} = 3$. So $\frac13\left.\frac{dy}{dx}\right|_{x=2} = 2\left\{\frac{1}{2+1} - \frac29\right\} = \frac29 \Rightarrow \left.\frac{dy}{dx}\right|_{x=2} = \underline{\underline{2/3}}$.

(c) $\frac{dy}{dx} = 1\cdot e^{1/x} + x\left\{e^{x^{-1}}(-x^{-2})\right\} = \left(1-\frac1x\right)e^{x^{-1}} \Rightarrow \left.\frac{dy}{dx}\right|_{x=-1/2} = \left(1+\frac{1}{1/2}\right)e^{-2} = \underline{\underline{\frac{3}{e^2}}}$

(d) $y = e^{\{x+2\sqrt{x}\}\ln(\ln(x))} \Rightarrow \frac{dy}{dx} = e^{\text{same}}\left\{\left(1+2\cdot\frac12 x^{-1/2}\right)\ln(\ln(x)) + \{x+2\sqrt{x}\}\frac{1}{\ln(x)}\cdot\frac1x\right\} \Rightarrow$

$\left.\frac{dy}{dx}\right|_{x=e} = e^{(e+2\sqrt{e})\ln(\ln(e))}\left\{\left(1+\frac{1}{\sqrt e}\right)\ln(\ln(e)) + (e+2\sqrt{e})\frac{1}{\ln(e)}\cdot\frac1e\right\}$

$= e^{(e+2\sqrt{e})\ln(1)}\left\{\left(1+\frac{1}{\sqrt e}\right)\ln(1) + \frac{e+2\sqrt e}{1\cdot e}\right\} = \underline{\underline{1+\frac{2}{\sqrt e}}}$ (because $\ln(1)=0$)

2 (a) $\frac{d}{dx}(x^2+4xy+y^2) = 0 \Rightarrow 2x + 4\{1\cdot y + xy'\} + 2yy' = 0 \Rightarrow y' = -\frac{(x+2y)}{2x+y}$

$\Rightarrow \left.\frac{dy}{dx}\right|_{(-1,5)} = -\frac{(-1+10)}{(-2+5)} = -3$. So the tangent line is $y-5 = -3(x-(-1))$ or $\underline{\underline{3x+y = 2}}$.

(b) From $(2x+y)y' = -x-2y$ we have, using the product rule)
$(2+y')y' + (2x+y)y'' = -1-2y'$. For $x=-1$, $y=5$ and hence
$y' = -3$ we have $(2-3)(-3) + \{-2+5\}y'' = -1+6 \Rightarrow \left.\frac{d^2y}{dx^2}\right|_{(-1,5)} = \frac{5-3}{3} = \frac23$

(c) Hence the hyperbola is concave up at $(-1,5)$.

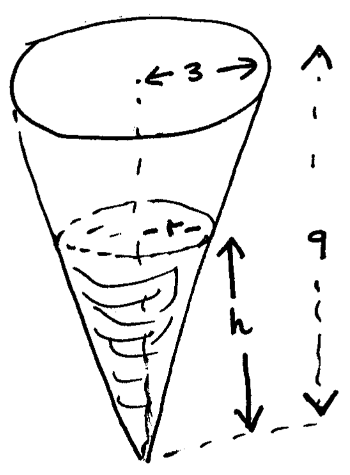
3. $f'(x) = 6x^2 - 6x - 12 + 0$
$= 6(x^2-x-2)$
$= 6(x+1)(x-2)$
$\Rightarrow$ critical points at $x = -1, 2$.

x	$f(x)$	TYPE
-3	-36	EP
-1	16	CP
2	-11	CP
3	0	EP

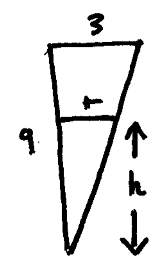
$\big(f(-3) = 2(-3)^3 - 3(-3)^2 - 12(-3) + 9 = -81+36+9 = -36$, etc.$\big)$

So, from the table, the unique absolute max & min are 16 at $x=-1$ and -36 at $x=-3$ respectively.

4.



Water volume is $V = \frac13\pi r^2 h$. But from similar triangles, $\frac{h}{r} = \frac93 = 3$



$\Rightarrow r = h/3$. So $V = \frac13\pi\left(\frac h3\right)^2 h = \frac{\pi h^3}{27}$

$\Rightarrow \frac{dV}{dt} = \frac{\pi}{27}\frac{d}{dt}(h^3) = \frac{\pi}{27}3h^2\cdot\frac{dh}{dt} = \frac{\pi h^2}{9}\frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt} = \frac{9}{\pi h^2}\frac{dV}{dt}$. So $\left.\frac{dh}{dt}\right|_{h=6} = \frac{9}{\pi\cdot6^2}\cdot 2$

$= \underline{\underline{\frac{1}{2\pi}}}$ cm/s (because $\frac{dV}{dt} = 2$ cm^3/s)