1. $f''(t) = 3t^{1/2} \Rightarrow f'(t) = 2t^{3/2} + B$; but $f'(4) = -1$, so $-1 = 2\,4^{3/2} + B \Rightarrow B = -17$. Now $f'(t) = 2t^{3/2} - 17 \Rightarrow f(t) = \frac{4}{5}t^{5/2} - 17t + C$; but $f(1) = \frac{3}{10}$, so $\frac{3}{10} = \frac{4}{5} - 17 + C \Rightarrow C = \frac{33}{2}$. So $f(t) = \underline{\underline{\frac{4}{5}t^{5/2} - 17t + \frac{33}{2}}}$

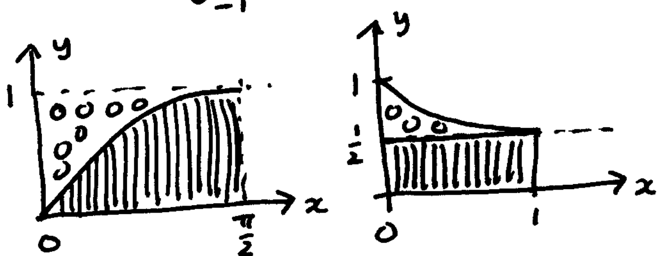
2. (a) With $u = x^2+1$, $\frac{d}{dx}\left[\ln(u^{1/2})\right] = \frac{d}{dx}\left[\frac{1}{2}\ln(u)\right] = \frac{1}{2}\frac{d}{dx}\{\ln(u)\} = \frac{1}{2}\frac{1}{u}\frac{du}{dx} = \frac{2x+0}{2u} = \underline{\underline{\frac{x}{x^2+1}}}$. So (b) $\int_1^{\sqrt{3}} \frac{1+4x}{x^2+1}\,dx = \int_1^{\sqrt{3}} \frac{1}{x^2+1}\,dx + 4\int_1^{\sqrt{3}} \frac{x}{x^2+1}\,dx = \arctan(x)\Big|_1^{\sqrt{3}} + 4\ln\left(\sqrt{x^2+1}\right)\Big|_1^{\sqrt{3}} = \arctan(\sqrt{3}) - \arctan(1) + 4\left\{\ln\left(\sqrt{3+1}\right) - \ln(\sqrt{2})\right\} = \pi/3 - \frac{\pi}{4} + 4\left\{\ln(2) - \frac{1}{2}\ln(2)\right\} = \underline{\underline{\frac{\pi}{12} + 2\ln(2)}}$

3. (a) $I = \int_1^4 \left(x^2 + \frac{1}{x} - 2\sqrt{x}\right)dx = \left\{\frac{1}{3}x^3 + \ln(x) - \frac{4}{3}x^{3/2}\right\}\Big|_1^4$
$= \frac{1}{3}4^3 + \ln(4) - \frac{4}{3}\cdot 4^{3/2} - \frac{1}{3} - \ln(1) + \frac{4}{3} = \underline{\underline{\frac{35}{3} + 2\ln(2)}}$

(b) $\cos(2x) > 0$ for $x \in [0, \pi/4]$, $\cos(2x) < 0$ for $x \in [\pi/4, \pi/2]$. So $I = \int_0^{\pi/4}\cos(2x)\,dx + \int_{\pi/4}^{\pi/2} -\cos(2x)\,dx =$
$\frac{1}{2}\sin(2x)\Big|_0^{\pi/4} - \frac{1}{2}\sin(2x)\Big|_{\pi/4}^{\pi/2} = \frac{1}{2}\sin\left(\frac{\pi}{2}\right) - \frac{1}{2}\sin(0) - \left\{\frac{1}{2}\sin(\pi) - \frac{1}{2}\sin\left(\frac{\pi}{2}\right)\right\}$
$= \frac{1}{2} - 0 - \left\{0 - \frac{1}{2}\right\} = 1$

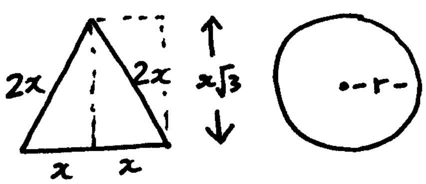
(c) $I = \int_{-1}^{1}(15x^2 + 17x - 4)\,dx = \left\{5x^3 + \frac{17}{2}x^2 - 4x\right\}_{-1}^{1} = \left(5 + \frac{17}{2} - 4\right) - \left(-5 + \frac{17}{2} + 4\right) = \underline{\underline{2}}$

4.



First region has area $\circ\circ = \frac{\pi}{2} - ||| = \frac{\pi}{2} - \int_0^{\pi/2}\sin(x)\,dx = \frac{\pi}{2} + \cos(x)\Big|_0^{\pi/2} = \frac{\pi}{2} + 0 - 1$. Second region has area $\circ\circ = \int_0^1 \frac{1}{x+1}\,dx - \frac{1}{2} = \ln(x+1)\Big|_0^1 - \frac{1}{2} = \ln(2) - \ln(1) - \frac{1}{2} = \ln(2) - \frac{1}{2}$. So the first area exceeds the second if $\frac{\pi}{2} - 1 > \ln(2) - \frac{1}{2}$ or $\pi > 2\ln(2) + 1$, which clearly holds because $\ln(2) < 1$ (because $e > 2$) and $\pi > 3$.

5.



Circle has circumference $8 - 6x$ and hence radius $r = \frac{8-6x}{2\pi} = \frac{4-3x}{\pi}$. So total enclosed area is
$A = x^2\sqrt{3} + \pi r^2 = \sqrt{3}x^2 + \frac{(4-3x)^2}{\pi} \Rightarrow \frac{dA}{dx} = 2\left\{x\sqrt{3} - \frac{3}{\pi}(4-3x)\right\} \Rightarrow$
$\frac{d^2A}{dx^2} = 2\left(\sqrt{3} + \frac{9}{\pi}\right) > 0$. So there is a minimum where $\frac{dA}{dx} = 0$ or $x = \frac{12}{9+\pi\sqrt{3}}$
So minimum total area is $\sqrt{3}\left(\frac{12}{9+\pi\sqrt{3}}\right)^2 + \frac{1}{\pi}\left\{4 - \frac{36}{9+\pi\sqrt{3}}\right\}^2 = \underline{\underline{\frac{16\sqrt{3}}{9+\pi\sqrt{3}}}} \approx 1.92$