

First Test

Thursday, September 16, 2004

You are allowed to use a TI-30Xa (or any four-function calculator). No other calculator is allowed. You have 75 minutes. Present your solutions clearly. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are stapled together in the proper order at the end of the test.

DO NOT WRITE ON THIS QUESTION PAPER, WHICH MUST BE TURNED IN AT THE END OF THE TEST (BUT NOT STAPLED TO YOUR SOLUTIONS)

1. Find each of the following limits:

$$(a) \lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1} \quad [4] \qquad (b) \lim_{x \rightarrow 0} \frac{3 \sin(2x) + 4x}{2 \sin(x) + 3x} \quad [4] \qquad (c) \lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4} \quad [4]$$

2. In each of the following cases, find $\frac{dy}{dx}$ from first principles:

$$(a) y = \frac{1}{x^2}, \quad x \neq 0 \quad [8] \qquad (b) y = \sqrt{3 + 2x^2} \quad [8]$$

3. (a) For $f(x) = 6x\{\pi \sin(x) + 3 \cos(x)\}$, find $f'(\frac{\pi}{3})$. [5]

(b) For $f(x) = \left\{ \frac{x+3}{x^2-3x+4} \right\}^4$, find $f'(1)$. [9]

4. A function is *smooth* if both it and its derivative are continuous everywhere. For what values of a and b is f defined by

$$f(x) = \begin{cases} a + b\sqrt{x} & \text{if } 0 \leq x \leq 1 \\ \frac{1}{x^2} & \text{if } 1 < x < \infty \end{cases}$$



a smooth function? [9]

Hint: Recycle the result you obtained for Question 2(a).

5. Find the equation of the tangent line to $y = (x + 3)\sqrt{3x^2 + 1}$ at the point $(4, 49)$. [9]