1 (a) $\lim_{x\to 1} \frac{x^3-1}{x^2-1} = \lim_{x\to 1} \frac{x^2+x+1}{x+1} = \frac{1^2+1+1}{1+1} = \frac{3}{2}$

(b) $\lim_{x\to 0} \frac{3\sin(2x)+4x}{2\sin(x)+3x} = \lim_{x\to 0} \frac{6\cdot\frac{\sin(2x)}{2x}+4}{2\frac{\sin(x)}{x}+3} = \frac{6\cdot 1+4}{2\cdot 1+3} = \frac{10}{5} = 2$

(c) $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} = \lim_{x\to 4} \frac{\sqrt{x}-2}{(\sqrt{x}-2)(\sqrt{x}+2)} = \lim_{x\to 4} \frac{1}{\sqrt{x}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{4}$

2 (a) $x^2y = 1$ and $(x+\delta x)^2(y+\delta y) = 1$ or $(x^2+2x\delta x+\delta x^2)(y+\delta y) = 1$
$\Rightarrow x^2\delta y + 2xy\,\delta x + o(\delta x) = 0$ after subtraction, because $\delta y \to 0$ as $\delta x \to 0$. So $x^2\frac{\delta y}{\delta x} + 2xy + \frac{o(\delta x)}{\delta x} = 0$. Letting $\delta x \to 0$ yields $x^2\frac{dy}{dx} + 2xy + 0 = 0$ or $\frac{dy}{dx} = -\frac{2xy}{x^2} = -\frac{2}{x^3}$ $(x \neq 0)$

(b) We have $y^2 = 3+2x^2$ (*) and $(y+\delta y)^2 = 3+2(x+\delta x)^2$ or $y^2+2y\delta y+\delta y^2 = 3+2\{x^2+2x\delta x+\delta x^2\}$. Subtraction of (*) yields $2y\delta y = 4x\delta x + o(\delta x)$ (because $\delta y \to 0$ as $\delta x \to 0$). Dividing by δx yields $2y\frac{\delta y}{\delta x} = 4x + \frac{o(\delta x)}{\delta x}$. Now letting $\delta x \to 0$ yields $2y\frac{dy}{dx} = 4x+0$ or $\frac{dy}{dx} = \frac{2x}{y} = \frac{2x}{\sqrt{3+2x^2}}$

3 (a) By the product rule, $f'(x) = \frac{d}{dx}(6x)\{\pi\sin(x)+3\cos(x)\} + 6x\frac{d}{dx}\{\pi\sin(x)+3\cos(x)\} = 6\{\pi\sin(x)+3\cos(x)\} + 6x\{\pi\cos(x)-3\sin(x)\}$. So $f'(\pi/3) = 6\{\pi\sin(\frac{\pi}{3})+3\cos(\frac{\pi}{3})\} + \frac{6\pi}{3}\{\pi\cos(\frac{\pi}{3})-3\sin(\pi/3)\} = 18\cos(\pi/3)+2\pi^2\cos(\pi/3) = 2\cos(\pi/3)\{9+\pi^2\} = 2\cdot\frac{1}{2}(9+\pi^2) = 9+\pi^2$
(because the two sine terms cancel in here).

(b) Set $y = f(x)$ and $w = \frac{x+3}{x^2-3x+4}$ so that $\frac{dw}{dx} = \frac{\frac{d}{dx}\{x+3\}(x^2-3x+4)-(x+3)\frac{d}{dx}(x^2-3x+4)}{(x^2-3x+4)^2} = \frac{(1+0)(x^2-3x+4)-(x+3)(2x-3+0)}{(x^2-3x+4)^2}$ by the quotient rule, and $y = w^4$, so that $\frac{dy}{dw} = 4w^3$.

Now $\frac{dy}{dx} = \frac{dy}{dw}\frac{dw}{dx} = 4w^3\frac{dw}{dx}$ and we obtain

$$f'(1) = \left.\frac{dy}{dx}\right|_{x=1} = \left.4w^3\frac{dw}{dx}\right|_{x=1} = 4\left(\frac{1+3}{1^2-3\cdot1+4}\right)^3 \left.\frac{dw}{dx}\right|_{x=1}$$

$$= 4\left(\frac{4}{2}\right)^3 \frac{1\cdot(1^2-3\cdot1+4)-(1+3)(2\cdot1-3)}{(1^2-3+4)^2} = \frac{4\cdot8\{2-(-4)\}}{2^2} = 48$$

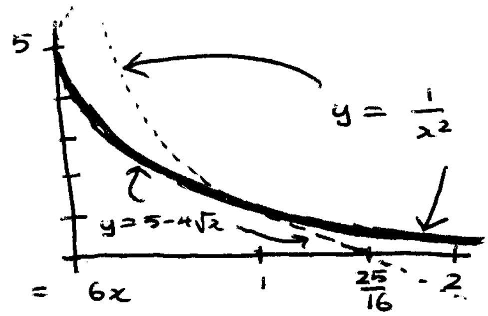
4 From the power rule, 2(a) and general properties:

$$f'(x) = \begin{cases} 0 + b\cdot\frac{1}{2}x^{-1/2} & \text{if } x\in[0,1) \\ -2/x^3 & \text{if } x\in(1,\infty). \end{cases}$$

Smoothness requires $f(1-) = f(1+)$ or $a + b\sqrt{1} = \frac{1}{1^2}$

and $f'(1-) = f'(1+)$ or $\frac{b}{2}1^{-1/2} = -\frac{2}{1^3}$.

So $b = -4$ and $a - 4 = 1 \Rightarrow a = 5$.



5. By the product rule, ⊙

$$\frac{dy}{dx} = \frac{d}{dx}(x+3)\sqrt{3x^2+1} + (x+3)\frac{d}{dx}\left[(3x^2+1)^{1/2}\right]$$

But with $u = 3x^2+1 \Rightarrow \frac{du}{dx} = 3\cdot2x+0 = 6x$

we have $\frac{d}{dx}\left[(3x^2+1)^{1/2}\right] = \frac{d}{dx}\left(u^{1/2}\right) = \frac{d}{du}\left(u^{1/2}\right)\frac{du}{dx} = \frac{1}{2}u^{-1/2}\frac{du}{dx}$

$$= \frac{1}{2}\frac{1}{\sqrt{3x^2+1}}\cdot 6x = \frac{3x}{\sqrt{3x^2+1}}.$$ Also, $\frac{d}{dx}(x+3) = 1+0 = 1$

So, from ⊙, $\frac{dy}{dx} = 1\cdot\sqrt{3x^2+1} + (x+3)\cdot\frac{3x}{\sqrt{3x^2+1}}$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=4} = \sqrt{3\cdot4^2+1} + \frac{7\cdot3\cdot4}{\sqrt{3\cdot4^2+1}}$$

$$= \sqrt{49} + \frac{7\cdot3\cdot4}{\sqrt{49}} = 7 + 12 = 19$$

So the tangent line has equation $y - 49 = 19(x-4)$ or $y = 19x - 27$.