1. For $t\in[0,1]$, $F(t)=\int_0^t\{7-2x\}\,dx = \{7x-x^2\}\big|_0^t = 7t-t^2 \Rightarrow F(1)=6$.
For $t\in(1,3]$, $F(t)=\int_0^t f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^t f(x)\,dx = F(1)+\int_1^t(2+3x^2)\,dx$
$= 6 + \{2x+x^3\}\big|_1^t = 6+2t+t^3-2\cdot1-1^3 = 3+2t+t^3$. Hence $F(t)=\begin{cases}7t-t^2 & \text{if } 0\le t\le 1\\ 3+2t+t^3 & \text{if } 1\le t\le 3\end{cases}$

2. (a) $g(0-)=g(0+) \Rightarrow 0^3+c=e^0 \Rightarrow c=1$. (b) $\int_{-1}^1 g(t)\,dt = \int_{-1}^0 g(t)\,dt + \int_0^1 g(t)\,dt =$
$\int_{-1}^0 (t^3+c)\,dt + \int_0^1 e^t\,dt = \{\frac{1}{4}t^4+ct\}\big|_{-1}^0 + e^t\big|_0^1 = \frac{1}{4}0^4 + c\cdot 0 - \frac{1}{4}(-1)^4 - c(-1) + e^1 - e^0 =$
$0+0-\frac{1}{4}+c+e-1 = e-\frac{1}{4}$ (≈ 2.468)

3. $f''(t) = 18t^{-3} + \frac{3}{2}t^{-3/2} + 2 \Rightarrow f'(t) = \int(18t^{-3}+\frac{3}{2}t^{-3/2}+2)\,dt = -9t^{-2}-3t^{-1/2}$
$+2t+b$, where b is an arbitrary constant. But $f'(1)=-10 \Rightarrow -9(1)^{-2}-3(1)^{-1/2}$
$+2\cdot1+b=-10 \Rightarrow b=0$. So $f'(t) = -9t^{-2}-3t^{-1/2}+2t \Rightarrow f(t) =$
$\int(-9t^{-2}-3t^{-1/2}+2t)\,dt = 9t^{-1}-6t^{1/2}+t^2+B$. But $f(1)=4 \Rightarrow 9-6+1$
$+B=4 \Rightarrow B=0$. So $f(t) = t^2-6\sqrt{t}+\frac{9}{t} = \left(t-\frac{3}{\sqrt{t}}\right)^2$.

4. $u=\sqrt[3]{21x+1} \Rightarrow u^3=21x+1 \Rightarrow x=\frac{u^3-1}{21} \Rightarrow \frac{dx}{du}=\frac{1}{21}(3u^2-0)=\frac{u^2}{7}$. So

$$I=\int_{x=0}^{x=3}\frac{35x+1}{\sqrt[3]{21x+1}}\,dx = \int_{u=\sqrt[3]{21\cdot0+1}}^{u=\sqrt[3]{21\cdot3+1}}\frac{35x+1}{\sqrt[3]{21x+1}}\frac{dx}{du}\,du = \int_{u=\sqrt[3]{1}}^{u=\sqrt[3]{64}}\frac{\frac{35}{21}(u^3-1)+1}{u}\,\frac{u^2}{7}\,du$$

$$=\int_1^4\left\{\frac{5}{3}(u^3-1)+1\right\}\frac{u}{7}\,du = \frac{1}{7}\int_1^4\left(\frac{5}{3}u^3-\frac{2}{3}\right)u\,du = \frac{1}{21}\int_1^4(5u^4-2u)\,du =$$

$$\frac{1}{21}(u^5-u^2)\Big|_1^4 = \frac{1}{21}\{4^5-4^2-0\} = \frac{1}{21}\{4^2(4^3-1)\} = \frac{1}{21}\cdot16\cdot63 = 48$$

5. (a) $I=\int_1^4\left\{x^2+2x\left(-\frac{3}{\sqrt{x}}\right)+\left(-\frac{3}{\sqrt{x}}\right)^2\right\}dx = \int_1^4\{x^2-6x^{1/2}+9x^{-1}\}\,dx$
$=\{\frac{1}{3}x^3-4x^{3/2}+9\ln(x)\}\big|_1^4 = \frac{64}{3}-4\cdot8+9\ln(4)-\frac{1}{3}+4-9\ln(1)$
$= 9\ln(4)-7 = 18\ln(2)-7$ (≈ 5.477)

(b) $I=\int_{-3}^3 x\{2x^2+x-1\}\,dx = \int_{-3}^3(2x^3+x^2-x)\,dx = 2\int_{-3}^3 x^3\,dx + \int_{-3}^3 x^2\,dx$
$-\int_{-3}^3 x\,dx = 2\cdot0 + 2\int_0^3 x^2\,dx - 0 = 2\,\frac{x^3}{3}\Big|_0^3 = 2\cdot(3^2-0) = 18$
because x^2 is even while x and x^3 are both odd.

(c) $I=\int_0^{\ln(3)}|e^x-3|\,dx + \int_{\ln(3)}^2|e^x-3|\,dx$

Note: $e^x>3 \Leftrightarrow \ln(e^x)>\ln(3) \Leftrightarrow x>\ln(3)$ and $2>\ln(3)$ because $e^2>3$

$=\int_0^{\ln(3)}(3-e^x)\,dx + \int_{\ln(3)}^2(e^x-3)\,dx$

$=\{3x-e^x\}\big|_0^{\ln(3)} + (e^x-3x)\big|_{\ln(3)}^2$

$=3\ln(3)-e^{\ln(3)}-0+e^0+e^2-6-e^{\ln(3)}+3\ln(3)$

$=3\ln(3)-3+1+e^2-6-3+3\ln(3)$

$=e^2-11+6\ln(3)$ (≈ 2.98)