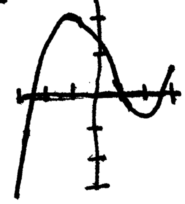
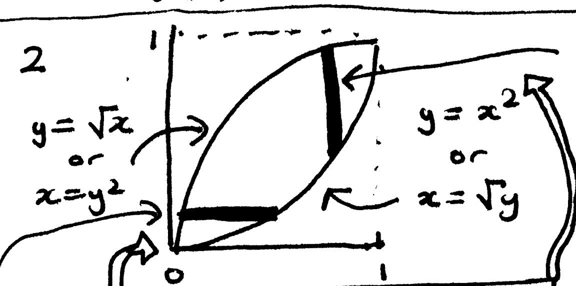
1(a) $f'(x) = (1-0)(2x^2 - x - 13) + (x-1)(4x - 1 - 0) = 6x^2 - 6x - 12$
$= 6(x+1)(x-2)$ and $f''(x) = 12x - 6 - 0 = 6(2x-1)$



$f'(-1) = 0$, $f''(-1) = -18 < 0 \Rightarrow -1$ is a local maximizer
$f'(2) = 0$, $f''(2) = 18 > 0 \Rightarrow 2$ is a local minimizer

(b) No corner extrema because f is smooth. $\therefore$ Only candidates for global extremizer are the critical points $-1, 2$ and the endpoints $-3, 3$. We have $f(-3) = (-4)(8) = -32$, $f(-1) = (-2)(-10) = 20$, $f(2) = 1.(-7) = -7$ and $f(3) = 2.2 = 4$. Hence $\max(f,-3,3) = f(-1) = 20$, $\min(f,-3,3) = f(-3) = -32$

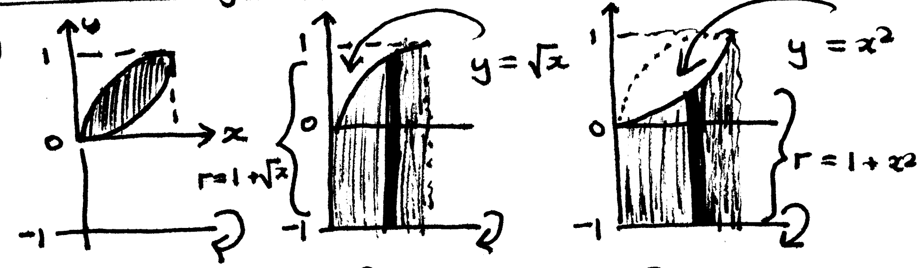
2



curves meet where $\sqrt{x} = x^2$
$\Rightarrow \sqrt{x}(1 - x\sqrt{x}) = 0 \Rightarrow \sqrt{x} = 0$ or
$x\sqrt{x} = 1 \Rightarrow x = 0$ or $x = 1$

With respect to x: $\delta A = (\sqrt{x} - x^2)\,\delta x + o(\delta x) \Rightarrow$
$A = \int_0^1 (x^{1/2} - x^2)\,dx = \left(\frac{2}{3}x^{3/2} - \frac{x^3}{3}\right)\Big|_0^1 =$
$\frac{2}{3}1^{3/2} - \frac{1^3}{3} - 0 = \frac{1}{3}$. With respect to y:
$\delta A = (\sqrt{y} - y^2)\,\delta y + o(\delta y) \Rightarrow A = \int_0^1 (y^{1/2} - y^2)\,dy$
$=$ same thing.

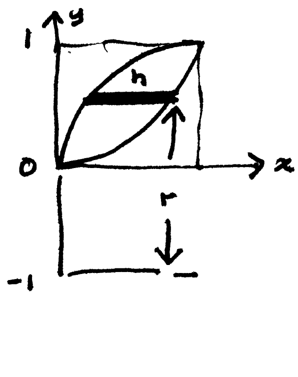
3(a)



Generates V | Generates V_2 | Generates V_1

For V_1, $\delta V = \pi r^2\,\delta x + o(\delta x)$
where $r = 1 + x^2 \Rightarrow$
$V_1 = \int_0^1 \pi r^2\,dx = \pi\int_0^1 (1+x^2)^2\,dx$
$= \pi\int_0^1 (1 + 2x^2 + x^4)\,dx = \pi\left\{x + \frac{2x^3}{3} + \frac{x^5}{5}\right\}\Big|_0^1$
$= \pi\left\{1 + \frac{2}{3} + \frac{1}{5} - 0\right\} = \frac{28\pi}{15}$. For V_2, $\delta V = \pi r^2\,\delta x + o(\delta x)$ where $r = 1 + x^{1/2} \Rightarrow$
$V_2 = \int_0^1 \pi r^2\,dx = \pi\int_0^1 (1 + x^{1/2})^2\,dx = \pi\int_0^1 (1 + 2x^{1/2} + x)\,dx = \pi\left\{x + \frac{4}{3}x^{3/2} + \frac{x^2}{2}\right\}\Big|_0^1$
$= \pi\left\{1 + \frac{4}{3} + \frac{1}{2} - 0\right\} = \frac{17\pi}{6}$. So $V = V_2 - V_1 = \frac{17\pi}{6} - \frac{28\pi}{15} = \frac{29\pi}{30}$

(b)



$t = y$
$h = \sqrt{y} - y^2$
$r = 1 + y$

$\delta V = 2\pi r h\,\delta t + o(\delta t)$
$= 2\pi(1+y)(y^{1/2} - y^2)\,\delta y + o(\delta y)$

$\Rightarrow V = \int_0^1 2\pi(1+y)(y^{1/2} - y^2)\,dy = 2\pi\int_0^1 \left\{y^{1/2} - y^2 + y^{3/2} - y^3\right\}dy$
$= 2\pi\left\{\frac{2}{3}y^{3/2} - \frac{y^3}{3} + \frac{2}{5}y^{5/2} - \frac{y^4}{4}\right\}\Big|_0^1 = 2\pi\left\{\frac{2}{3} - \frac{1}{3} + \frac{2}{5} - \frac{1}{4} - 0\right\}$
$= 2\pi\left\{\frac{1}{3} + \frac{2}{5} - \frac{1}{4}\right\} = 2\pi\left\{\frac{20 + 24 - 15}{60}\right\} = \frac{29\pi}{30}$.

4. Set $g(x) = 1 - \cos 2x \Rightarrow g'(x) = 0 - (-\sin(2x)\cdot 2) = 2\sin(2x)$
$\Rightarrow g''(x) = 2\cdot 2\cos(2x) = 4\cos(2x)$ and
$h(x) = \sin(x) - \ln(1+x) \Rightarrow h'(x) = \cos(x) - \frac{1}{1+x}(0+1) =$

$\cos(x) - (1+x)^{-1} \Rightarrow h''(x) = -\sin(x) - \{-(1+x)^{-2}(0+1)\} = -\sin(x) + \dfrac{1}{(1+x)^2}$

So $g(0) = 1-\cos(0) = 0$, $g'(0) = 2\sin(0) = 0$, $g''(0) = 4\cos(0) = 4$

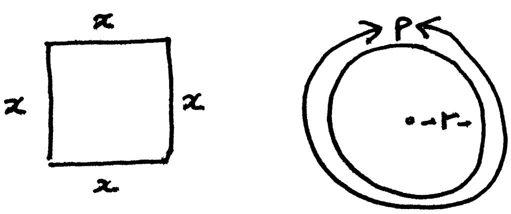
and $h(0) = \sin(0) - \ln(1) = 0$, $h'(0) = \cos(0) - \dfrac{1}{1+0} = 0$, $h''(0) = -\sin(0) + \dfrac{1}{(1+0)^2} = 1$

$$\Rightarrow \lim_{x\to 0} \frac{g(x)}{h(x)} = \frac{g''(0)}{h''(0)} = \frac{4}{1} = 4 \quad \text{by L'Hopital's rule.}$$

(However, I still prefer to use $\cos(u) = 1 - \frac{1}{2}u^2 + O(u^4)$, $\sin(u) = u + O(u^3)$ and $\ln(1+u) = u - \frac{1}{2}u^2 + O(u^3)$ to obtain the result by letting $x \to 0$ in

$$\frac{1-\cos(2x)}{\sin(x) - \ln(1+x)} = \frac{1 - \{1 - \frac{1}{2}(2x)^2 + O(x^4)\}}{x + O(x^3) - x + \frac{1}{2}x^2 + O(x^3)} = \frac{2x^2 + O(x^4)}{\frac{1}{2}x^2 + O(x^3)} = \frac{2 + O(x^2)}{\frac{1}{2} + O(x)}$$)

5.



We have $4x + P = L$ and

$$2\pi r = P$$

$$\Rightarrow r = \frac{P}{2\pi} = \frac{L-4x}{2\pi}$$

Hence square has area $S = x^2$

circle has area $C = \pi r^2 = \dfrac{(L-4x)^2}{4\pi}$

and total area is

$$A = S + C = x^2 + \frac{1}{4\pi}(L-4x)^2$$

$$\Rightarrow \frac{dA}{dx} = 2x + \frac{1}{4\pi}\cdot 2(L-4x)(-4) = 2\left\{x - \frac{(L-4x)}{\pi}\right\}$$

$$= \frac{2}{\pi}\{(\pi+4)x - L\}$$

$$\Rightarrow \frac{d^2A}{dx^2} = \frac{2(\pi+4)}{\pi}.1 - 0 = 2\left(1 + \frac{4}{\pi}\right) > 0$$

So there must be a minimum where $\dfrac{dA}{dx} = 0 \Rightarrow$

(a) $x = \dfrac{L}{\pi+4} \Rightarrow S = \left(\dfrac{L}{\pi+4}\right)^2$

Then (b) $r = \dfrac{L-4x}{2\pi} = \dfrac{L}{2\pi}\left\{1 - \dfrac{4}{\pi+4}\right\} = \dfrac{L}{2(\pi+4)}$

$\Rightarrow C = \dfrac{\pi}{4}\left(\dfrac{L}{\pi+4}\right)^2 \Rightarrow$ square has larger area ($\pi < 4$) and

(c) ratio $= \dfrac{S}{C} = \dfrac{4}{\pi}$

Note that diameter of circle = side of square in the optimal configuration.