MAC 2311 Section 07, Fall 2004 (Dr M-G)

Final

Monday, December 06, 2004

You are allowed to use a TI-30Xa (or any four-function calculator). No other calculator is allowed. You have 75 minutes. Present your solutions clearly. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are stapled together in the proper order at the end of the test.

DO NOT WRITE ON THIS QUESTION PAPER, WHICH MUST BE TURNED IN AT THE END OF THE TEST (BUT NOT STAPLED TO YOUR SOLUTIONS)

1. Find an equation of the tangent line to $y = (x^3+1)\sin\left(\frac{1}{2}\pi x\right)$ at the point $(1, 2)$. [12]

2. **(a)** If $f''(t) = \frac{4}{t^3}$ for all $t > 0$, $f'(1) = -1$ and $f(1) = 3$, find $f(t)$ exactly. [12]

(b) Use the substitution $u = \sqrt{5x+1}$ to find the exact value of $I = \int_0^3 \frac{5x-1}{\sqrt{5x+1}}\,dx$. [12]

(c) Find the exact value of $I = \int_0^1 |e^x - 2|\,dx$. [6]

3. Find both $m = \left.\frac{dy}{dx}\right|_{(x,y)=(1,-1)}$ and $\alpha = \left.\frac{d^2y}{dx^2}\right|_{(x,y)=(1,-1)}$ for $x - y + x^3 - y^3 = 4$. [12]

4. If R is the region enclosed by $y = 0$, $y = x + \frac{3}{\sqrt{x}}$, $x = 1$ and $x = 4$:

(a) Find the area of R. [10]

(b) Find the volume generated by rotating R about the y-axis. [10]

(c) Find the volume generated by rotating R about the x-axis. [10]

5. Use L'Hôpital's rule to calculate $\lim\limits_{x\to 0} \frac{1-\cos(2x)}{\ln(1+3x^2)}$ [12]

6. Find the global extrema of f defined on $[4, 10]$ by $f(x) = \ln\left(\frac{7}{x}\right) - \frac{6}{x}$. You may assume that $\ln\left(\frac{5}{2}\right) > \frac{9}{10}$. [12]

7. A tank in the shape of an inverted cone of height h and maximum radius r is filled with liquid of density ρ. If g is the acceleration due to gravity, how much work is required to pump all of the liquid out again? [12]

[Perfect score: $12 + 30 + 12 + 30 + 3 \times 12 = 120$]