

Final

Monday, December 06, 2004

**You are allowed to use a TI-30Xa (or any four-function calculator). No other calculator is allowed. You have 75 minutes. Present your solutions clearly. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are stapled together in the proper order at the end of the test.**

**DO NOT WRITE ON THIS QUESTION PAPER, WHICH MUST BE TURNED IN AT THE END OF THE TEST (BUT NOT STAPLED TO YOUR SOLUTIONS)**

1. Find the equation of the tangent line to  $y = \sqrt{x^2 + 3} e^x$  at the point  $(1, 2e)$ . [12]
2. If  $R$  is the region enclosed by  $y = 0$ ,  $y = 2x + \frac{1}{x}$ ,  $x = \frac{1}{2}$  and  $x = 1$ :
  - (a) Find the area of  $R$ . [10]
  - (b) Find the volume generated by rotating  $R$  about the  $y$ -axis. [10]
  - (c) Find the volume generated by rotating  $R$  about the  $x$ -axis. [10]
3. A ladder 3 meters long rests against a vertical wall. If the top of the ladder slides down the wall at a rate of  $\frac{1}{4}$  meter per second, how fast is the bottom of the ladder sliding away from the wall when it is  $\frac{1}{2}$  meter from the wall? [12]
4. Use L'Hôpital's rule to calculate  $\lim_{x \rightarrow 0} \frac{\cos(3x) - e^x + \ln(1+x)}{\sin(x) + \ln(1-x)}$  [12]
5. (a) If  $f''(t) = -\frac{2}{t^3}$  for all  $t > 0$ ,  $f'(1) = 3$  and  $f(1) = 1$ , find  $f(t)$  exactly. [12]
  - (b) Use the substitution  $u = \sqrt{4x+1}$  to find the exact value of  $I = \int_0^2 \frac{3x-1}{\sqrt{4x+1}} dx$ . [12]
  - (c) Find the exact value of  $I = \int_{-\pi/2}^{\pi/2} |\sin(x) + 4x| dx$ . [6]
6. What is the largest possible volume for an open box with a square base whose total surface area is 48 square meters? [12]
7. A hemi-spherical tank of radius  $r$  is full of liquid of density  $\rho$ . If  $g$  is the acceleration due to gravity, how much work is required to pump all the liquid out? [12]