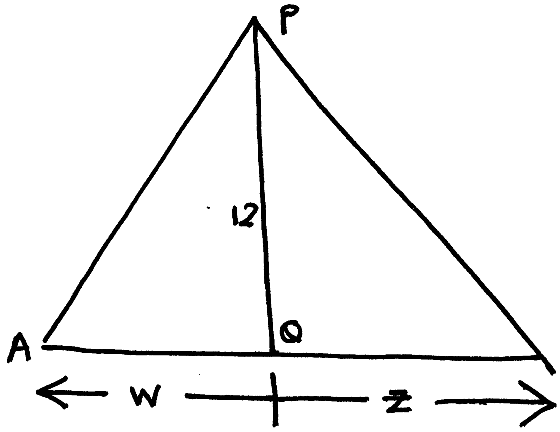


Greek letter zeta (as in Catherine Zeta-Jones)

Given : $\dfrac{dw}{dt} = 2$ and $AP + PB = 39$

$$\Rightarrow \sqrt{w^2+12^2} + \sqrt{z^2+12^2} = 39 \quad (*) \qquad \text{(by Pythagoras)}$$

Want to find : rate at which BQ <u>decreases</u> when $w = 5$

$$= -\left.\frac{dz}{dt}\right|_{w=5} = \zeta, \text{ say.}$$

Differentiating $(*)$: $\dfrac{d}{dt}\sqrt{w^2+12^2} + \dfrac{d}{dt}\sqrt{z^2+12^2} = 0$

$$\Rightarrow \frac{d}{dw}\left[\sqrt{w^2+12^2}\right]\frac{dw}{dt} + \frac{d}{dz}\left[\sqrt{z^2+12^2}\right]\frac{dz}{dt} = 0$$

$$\Rightarrow \frac{w}{\sqrt{w^2+12^2}}\frac{dw}{dt} + \frac{z}{\sqrt{z^2+12^2}}\frac{dz}{dt} = 0 \quad (**)$$

on using

When $w = 5$ we have $AP = \sqrt{5^2+12^2} = 13$

$$\Rightarrow PB = 39 - AP = 39 - 13 = 26$$

$$\Rightarrow z = \sqrt{PB^2 - 12^2} = \sqrt{26^2 - 12^2} = \sqrt{2^2(13^2-6^2)}$$

$= 2\sqrt{13^2-6^2} = 2\sqrt{133}$. So, on setting $w = 5$ in $(**)$, we have

$$0 = \left.\frac{w}{\sqrt{w^2+12^2}}\right|_{w=5}\left.\frac{dw}{dt}\right|_{w=5} + \left.\frac{z}{\sqrt{z^2+12^2}}\right|_{w=5}\left.\frac{dz}{dt}\right|_{w=5}$$

$$= \frac{5}{13}\cdot 2 + \frac{2\sqrt{133}}{26}(-\zeta)$$

$$\Rightarrow \zeta = \frac{10}{\sqrt{133}} \approx 0.867 \text{ ft/s}$$

SEPARATE CALCULATION

What is $\dfrac{d}{dx}\left[\sqrt{x^2+12}\right]$?

Set $y = \sqrt{x^2+12}$

$\Rightarrow y^2 = x^2+12$

$\Rightarrow \dfrac{d}{dx}(y^2) = 2x + 0$

$\Rightarrow 2y\dfrac{dy}{dx} = 2x$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{x}{y} = \dfrac{x}{\sqrt{x^2+12^2}}$

OR

Put $u = x^2+12$

$\Rightarrow y = u^{1/2}$

$\Rightarrow \dfrac{dy}{du} = \dfrac{1}{2}u^{-1/2}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

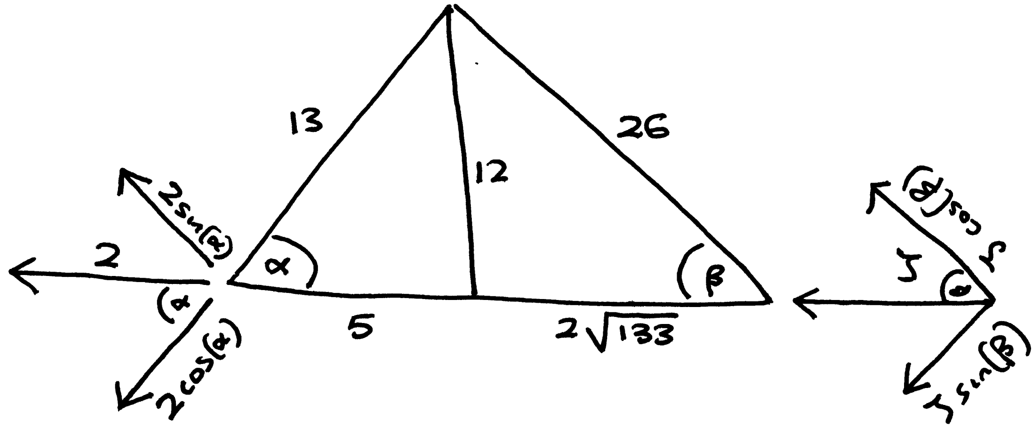
$= \dfrac{1}{2}u^{-1/2}\cdot 2x$

$= \dfrac{x}{\sqrt{u}}$

$= \dfrac{x}{\sqrt{x^2+12^2}}$

as before

Alternatively, resolve the velocities of the carts at the instant in question along and perpendicular to the rope:



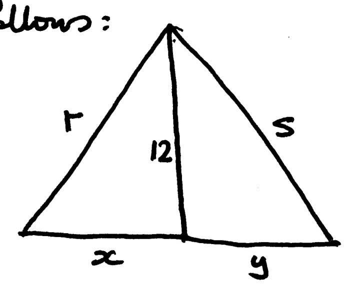
Since the carts are attached to the rope,

A's velocity in direction of rope = velocity of rope = B's velocity in direction of rope

$$\Rightarrow \quad 2\cos(\alpha) = J\cos\beta$$

$$\Rightarrow \quad J = \frac{2\cos(\alpha)}{\cos(\beta)} = 2\,\frac{\frac{5}{13}}{\frac{2\sqrt{133}}{26}} = \frac{10}{\sqrt{133}} \quad \text{as before}$$

Yet another possibility would be to proceed as follows:



$$r^2 = 12^2 + x^2 \Rightarrow 2r\frac{dr}{dt} = 0 + 2x\frac{dx}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{x}{r}\frac{dx}{dt} \Rightarrow \left.\frac{dr}{dt}\right|_{x=5} = \frac{5}{13}\cdot 2 = \frac{10}{13}$$

$$s^2 = y^2 + 12^2 \Rightarrow 2s\frac{ds}{dt} = 2y\frac{dy}{dt}$$

$$\Rightarrow J = -\left.\frac{dy}{dt}\right|_{x=5} = -\frac{s}{y}\left.\frac{ds}{dt}\right|_{x=5} = \frac{-26}{2\sqrt{133}}\left.\frac{ds}{dt}\right|_{x=5} = \frac{-13}{\sqrt{133}}\left.\frac{ds}{dt}\right|_{x=5}$$

But $r + s = 39 \Rightarrow \frac{dr}{dt} + \frac{ds}{dt} = 0 \Rightarrow \frac{ds}{dt} = -\frac{dr}{dt}$. So

$$J = \frac{-13}{\sqrt{133}}\left(-\left.\frac{dr}{dt}\right|_{x=5}\right) = \frac{-13}{\sqrt{133}}\left(-\frac{10}{13}\right) = \frac{10}{\sqrt{133}} \quad \text{as before.}$$

Of course, the second method requires no calculus (only trigonometry), demonstrating that sometimes oldies are goldies.