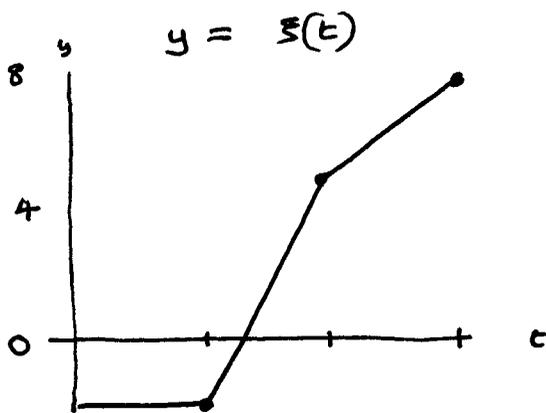


ξ is piecewise-linear, hence the only possible discontinuity is at $t=1$ or $t=2$. But $\xi(1^-) = \lim_{t \rightarrow 1^-} \xi(t) = -2$ and $\xi(1^+) = \lim_{t \rightarrow 1^+} \xi(t) = 7 \cdot 1 - 9 = -2 \Rightarrow \xi(1^-) = \xi(1^+) \Rightarrow \xi$ continuous at $t=1$; and similarly, $\xi(2^-) = 7 \cdot 2 - 9 = 5 = 3 \cdot 2 - 1 = \xi(2^+) \Rightarrow \xi$ continuous at $t=2$. So ξ is continuous.

$$\underline{0 \leq t \leq 1}: \phi(t) = \int_0^t \xi(x) dx = \int_0^t -2 dx = \int_0^t -2 \cdot 1 dx$$

$$= -2 \int_0^t 1 dx = -2(t-0) = -2t. \text{ In particular, } \phi(1) = -2$$

$$\underline{1 \leq t \leq 2}: \phi(t) = \int_0^t \xi(x) dx = \int_0^1 \xi(x) dx + \int_1^t \xi(x) dx$$

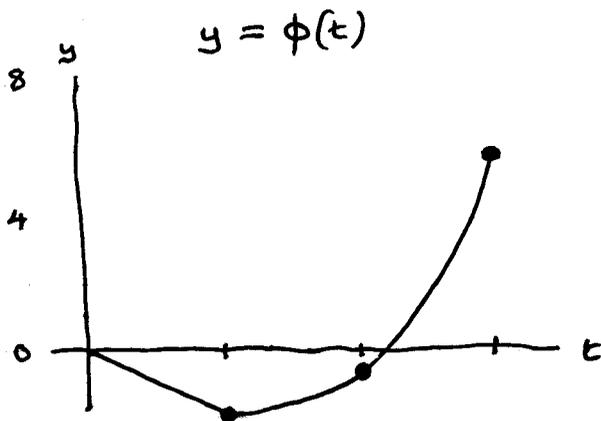


$$= \phi(1) + \int_1^t (7x - 9) dx$$

$$= -2 + 7 \int_1^t x dx - 9 \int_1^t 1 dx$$

$$= -2 + 7 \left(\frac{t^2 - 1^2}{2} \right) - 9(t-1)$$

$$= \frac{7}{2} t^2 - 9t + \frac{7}{2}. \text{ In particular, } \phi(2) = \frac{7}{2} \cdot 2^2 - 9 \cdot 2 + \frac{7}{2} = -\frac{1}{2}.$$



Finally:

$$\underline{2 \leq t \leq 3}: \phi(t) = \int_0^t \xi(x) dx$$

$$= \int_0^2 \xi(x) dx + \int_2^t \xi(x) dx$$

$$= \phi(2) + \int_2^t (3x - 1) dx$$

$$= -\frac{1}{2} + 3 \int_2^t x dx - \int_2^t 1 dx$$

$$= -\frac{1}{2} + 3 \left(\frac{t^2 - 2^2}{2} \right) - (t-2) = \frac{3}{2} t^2 - t - \frac{9}{2}$$

$$\text{So } \phi(t) = \begin{cases} -2t & \text{if } 0 \leq t \leq 1 \\ \frac{7}{2} t^2 - 9t + \frac{7}{2} & \text{if } 1 \leq t \leq 2 \\ \frac{3}{2} t^2 - t - \frac{9}{2} & \text{if } 2 \leq t \leq 3 \end{cases}$$