MAC 2311, Sections 09 & 10, Spring 2005 (Dr M-G)

Mock Third Test

Tuesday, March 29, 2005

You are allowed to use a TI-30Xa/TI-36X (or any four-function calculator). No other calculator is allowed. You have 75 minutes. Present your solutions clearly. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are stapled together in the proper order at the end of the test.

DO NOT WRITE ON THIS QUESTION PAPER, WHICH MUST BE TURNED IN AT THE END OF THE TEST (BUT NOT STAPLED TO YOUR SOLUTIONS)

1. A function f is defined on $[-15, 15]$ by

$$f(x) \;=\; \begin{cases} \dfrac{33x}{16+x} + 657 & \text{if} \quad -15 \le x < -5 \\ 17 + 60x + 27x^2 - 2x^3 & \text{if} \quad -5 \le x < 12 \\ \dfrac{16x}{16-x} + 1121 & \text{if} \quad 12 \le x \le 15. \end{cases}$$

(a) Verify that f is continuous. [2]
(b) Find expressions for $f'(x)$ and $f''(x)$. [6]
(c) Find and identify all local extrema. [4]
(d) Where is f concave up? Concave down? Find all inflection points. [4]
(e) Find all global extrema of f on $[-15, 15]$. [2]

2. Functions G and g are defined on $[0, 3]$ by $G(t) = \int\limits_0^t g(x)\,dx$ and

$$g(t) \;=\; \begin{cases} 4(5-t) & \text{if} \quad 0 \le t < 2 \\ t^2 - t^3 + t^4 & \text{if} \quad 2 \le t \le 3. \end{cases}$$

(a) Verify that g is continuous on $[0, 3]$ [2]
(b) Assuming that $\int_a^b x^s\,dx = \frac{b^{s+1} - a^{s+1}}{s+1}$ for $0 \le s \le 4$ but *without invoking the fundamental theorem of calculus*, find an explicit formula for $G(t)$ for all $t \in [0, 3]$. [6]

3. In each of the following cases, find the exact value of the definite integral:

(a) $I = \int\limits_1^2 \left\{ x - \dfrac{2}{\sqrt{x}} \right\}^3 dx$ [7] **(b)** $I = \int\limits_{-2}^1 (3-4x)(5-2x)\,dx$ [4] **(c)** $I = \int\limits_0^1 |e^{2x} - 3|\,dx$ [7]

4. Find $F'(t)$ for $F(t) = \int\limits_{t^3}^{-4} \sqrt{1+x^4}\,dx$. [6]

5. Any optimization problem. One of many possibilities would be the following: A square cookie cutter and a circular cookie cutter are to be made from a thin strip of metal of length L by cutting it into two pieces and bending each piece appropriately. If the sum of the areas of the two resulting cookies must be as small as possible then

(a) How long is a side of the square cookie? [6]
(b) What is the radius of the circular cookie? [2]
(c) What is the ratio of the larger cookie's area to the area of the smaller one? [2]

Another would be the following: What are (i) the maximum volume of an open-topped right-circular cylinder whose surface area is S and (ii) the dimensions (radius and height) that achieve it? [10]

[Perfect score: $18 + 8 + 18 + 6 + 10 = 60$]