First Test
Tuesday, February 1, 2005

You are allowed to use a TI-30Xa (or any four-function calculator). No other calculator is allowed. You have 75 minutes. Present your solutions clearly. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are stapled together in the proper order at the end of the test.

DO **NOT** WRITE ON THIS QUESTION PAPER, WHICH MUST BE TURNED IN AT THE END OF THE TEST (BUT **NOT** STAPLED TO YOUR SOLUTIONS)

1. In each of the following cases, find $\frac{dy}{dx}$ *from first principles* (i.e., by using the definition of differential coefficient or derivative in terms of a limit):

    (a) $y = x^5$ [5]   **(b)** $y = \dfrac{8}{x}$ (where $x \neq 0$)   [5]

2. In each of the following cases, find the *exact* value of $f'(1)$:

    (a) $f(x) = 2x^3 \sin\left(\frac{1}{4}\pi x\right) + \cos\left(\frac{1}{4}\pi x\right)$ [6]

    (b) $f(x) = \left\{ \frac{1+x}{1-x-x^2} \right\}^5$ (where $x > \frac{\sqrt{5}-1}{2}$)   [9]

3. A function is *smooth* if both it and its derivative are continuous everywhere. For what values of a and b is f defined by

$$f(x) = \begin{cases} a\,x + b\,x^3 & \text{if} \quad 0 \leq x \leq 2 \\ \dfrac{8}{x} & \text{if} \quad 2 < x < \infty \end{cases}$$

a smooth function?   [10]

Hint: Recycle the result you obtained for Question **1(b)**.

4. **(a)** Find the equation of the tangent line to the curve $y = (x^2 + 4)\sqrt{x + 2}$ at the point $(2, 16)$ [9]

    (b) Verify that the tangent line passes through the point $(1, 6)$.   [1]

[Perfect score: $10 + 15 + 2 \times 10 = 45$]