1. $\ln(y) = \ln(3x+1) + \ln(e^{x^2}) - \ln(x+3)$

$= \ln(3x+1) + x^2 - \ln(x+3)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{3x+1}\cdot 3 + 2x - \frac{1}{x+3}\cdot 1$ Put $x=1, y=e$

and $m = \left.\frac{dy}{dx}\right|_{x=1}$ to get $\frac{1}{e}\cdot m = \frac{1}{3+1}\cdot 3 + 2 - \frac{1}{4} = \frac{5}{2}$ or

$m = \frac{5}{2}e$. So the tangent line is $y - e = \frac{5}{2}e(x-1)$ or $y = \frac{1}{2}e(5x-3)$

2. $\frac{d}{dx}(2x^3) + \frac{d}{dx}(y^4) + \frac{d}{dx}(2) = \frac{dy}{dx} \Rightarrow 2\cdot 3x^2 + 4y^3\frac{dy}{dx} + 0 = \frac{dy}{dx}$ (*)

Differentiate again: $6\frac{d}{dx}(x^2) + 4\frac{d}{dx}\left(y^3\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} \Rightarrow$

$6\cdot 2x + 4\left\{\frac{d}{dx}(y^3)\frac{dy}{dx} + y^3\frac{d^2y}{dx^2}\right\} = \frac{d^2y}{dx^2} \Rightarrow$

$12x + 4\left\{3y^2\left(\frac{dy}{dx}\right)^2 + y^3\frac{d^2y}{dx^2}\right\} = \frac{d^2y}{dx^2}$ (**) Now put $x=-1, y=1$

in (*) to get $2\cdot 3(-1)^2 + 4\cdot 1^3 m = m$ or $m = -2$. Now put

$x=-1, y=1$ in (**) to get $12(-1) + 4\{3\cdot 1^2 m^2 + 1^3\alpha\} = \alpha$ or

$3\alpha = 12 - 12m^2 \Rightarrow \alpha = 4 - 4m^2 = -12$

3. (a) $e^y = 1 + e^{3x} \Rightarrow \frac{d}{dx}(e^y) = 0 + e^{3x}\frac{d}{dx}(3x) \Rightarrow$

$e^y\frac{dy}{dx} = e^{3x}\cdot 3$. Now put $x=0 \Rightarrow y = \ln(1+e^0)$

$= \ln(2) \Rightarrow e^{\ln(2)}\cdot m = e^0\cdot 3 \Rightarrow m = 3/2$

(b) $\ln\left(\sqrt{x^2+6x+3}\right) = \ln\left((x^2+6x+3)^{1/2}\right) = \frac{1}{2}\ln(x^2+6x+3)$.

Also, $\frac{d}{du}\{\sinh^{-1}(u)\} = \frac{1}{\sqrt{u^2+1}} \Rightarrow \frac{d}{dx}\{\sinh^{-1}(5x)\} = \frac{1}{\sqrt{(5x)^2+1}}\frac{d}{dx}(5x)$

$= \frac{1}{\sqrt{25x^2+1}}\cdot 5$. So $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\{\ln(w)\} + \frac{5}{\sqrt{25x^2+1}}$

$= \frac{1}{2}\frac{1}{w}\frac{dw}{dx} + \frac{5}{\sqrt{25x^2+1}}$

where $w = x^2+6x+3 \Rightarrow \frac{dw}{dx} = 2x+6$. That is,

$\frac{dy}{dx} = \frac{x+3}{x^2+6x+3} + \frac{5}{\sqrt{25x^2+1}}$. Now put $x=0$ to

obtain $m = \frac{0+3}{0+3} + \frac{5}{\sqrt{0+1}} = 1+5 = 6$

4 (a) $$\frac{dx}{dt} = \frac{\frac{d}{dt}\left[\ln(1+t)\right]\cdot(1+t) - \ln(1+t)\cdot(0+1)}{(1+t)^2}$$

$$= \frac{\frac{1}{1+t}\, 1+t - \ln(1+t)}{(1+t)^2} = \frac{1 - \ln(1+t)}{(1+t)^2}.$$

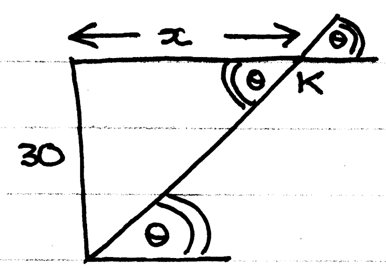
(b) $$\frac{d^2x}{dt^2} = \frac{\left(0 - \frac{1}{1+t}\right)\cdot(1+t)^2 - \left(1 - \ln(1+t)\right) 2(1+t)}{(1+t)^4}$$

$$= \frac{2\ln(1+t) - 3}{(1+t)^3}$$

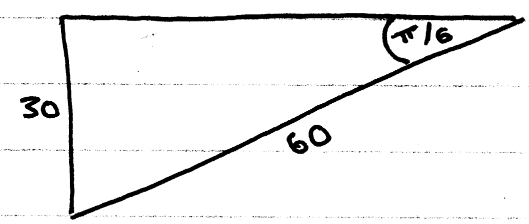
(c) $$\frac{dx}{dt} = 0 \Rightarrow \ln(1+t) = 1 \Rightarrow 1+t = e^1 \Rightarrow t = e-1$$

(d) $$\frac{d^2x}{dt^2} = 0 \Rightarrow \ln(1+t) = \frac{3}{2} \Rightarrow 1+t = e^{3/2} \Rightarrow t = e\sqrt{e}-1$$

5.



MOVIE



STILL

$$\frac{30}{x} = \tan(\theta)$$ (turn page upside-down if necessary)

$$\Rightarrow x = 30\cot(\theta)$$

$$\Rightarrow \frac{dx}{dt} = 30\frac{d}{dt}\left[\cot(\theta)\right] = 30\frac{d}{d\theta}\left[\cot(\theta)\right]\frac{d\theta}{dt}$$

$$= -\frac{30}{\sin^2(\theta)}\frac{d\theta}{dt}$$ (using the given result)

So $$\frac{d\theta}{dt} = -\frac{1}{30}\sin^2(\theta)\frac{dx}{dt} = \frac{-1}{30}\sin^2(\theta)\cdot 3$$

$$= -\frac{1}{10}\sin^2(\theta) \Rightarrow$$

$$-\frac{d\theta}{dt}\bigg|_{\theta=\pi/6} = -\left(-\frac{1}{10}\sin^2\left(\frac{\pi}{6}\right)\right) = \frac{1}{10}\left(\frac{1}{2}\right)^2 = \frac{1}{40} \text{ rad/s}$$